New College Bradford
A-Level Mathematics
Y12-13
Summer Independent Learning

	Content	Answers
Consolidation	Partial Fractions Differentiation Forces at Angles Modulus Function Trapezium Rule	Partial Fractions Differentiation Forces at Angles Modulus Function Trapezium Rule
Review	Pure Paper Mechanics Paper	Pure Paper Mechanics Paper
Extension	Optional Extension	

The sections highlighted in yellow are **compulsory** and need to be completed and marked. Then you will hand in the work during your first maths lesson in Year 13.

The rest are **optional** and are meant to extend your mathematical knowledge.

Consolidation: Partial Fractions

Knowledge Check

1. By substituting appropriate values of x, work out the values of the constants in these identities.

a $7x-15 \equiv A(x-1)+B(x-3)$

b $4x^2+24x+15 \equiv C(x+2)+D(x+2)^2 + E(x+1)$

c $24x-24 \equiv Fx(x+4)+G(x+4)(x-2) + Hx(x-2)$

2. Express each of these using partial fractions.

a $\dfrac{x-17}{(x+3)(x-2)}$ **b** $\dfrac{10x-2}{(x-1)(x+1)^2}$

c $\dfrac{146-38x}{(2x-5)(x+6)(2x+1)}$

3. **Express these improper** fractions as partial fractions

a $\dfrac{3x^2-10x+11}{(x-1)(x-3)}$ **b** $\dfrac{5x^2+27x+26}{(x+1)(x+5)}$

Reasoning

A student tries to decompose $\dfrac{14}{(x-5)(x+1)^2}$ into partial fractions. Explain and correct her mistakes.

$$\frac{14}{(x-5)(x+1)^2} \equiv \frac{A}{(x-5)} + \frac{B}{(x+1)^2}$$

$$14 \equiv A(x+1)^2 + B(x-5)$$

Let $x=-1 \Rightarrow 14 \equiv 0A-6B$ so $B=-\dfrac{7}{3}$

Let $x=5 \Rightarrow 14 \equiv 36A+0B$ so $A=\dfrac{7}{18}$

$$\frac{14}{(x-5)(x+1)^2} \equiv \frac{7}{18(x-5)} - \frac{7}{3(x+1)^2}$$

Challenge

Given that a, b, c, P and Q are constants, write expressions for P and Q in terms of a, b and c if $\dfrac{ax+b}{(x+c)^2} = \dfrac{P}{x+c} + \dfrac{Q}{(x+c)^2}$

Consolidation: Differentiation

Knowledge Check

1. Differentiate each of these functions.

- **a** $(3x+4)^5$
- **b** $(2x-1)^7$
- **c** $(x^2+1)^6$
- **d** $(1-2x-3x^2)^3$
- **e** $\sqrt{2x+1}$
- **f** $\sqrt{3-5x}$
- **g** $\sqrt{3x^2+4}$
- **h** $\sqrt[3]{1-2x}$
- **i** $\sqrt{x^2+3x+4}$
- **j** $\frac{1}{(2x+3)^2}$
- **k** $\frac{1}{\sqrt{1-3x}}$
- **l** $\frac{3}{(x^2-2x+5)}$
- **m** $\cos^2 x$
- **n** $\sqrt{\cos x}$
- **o** $\sin(3x+2)$
- **p** $\tan(5x-1)$
- **q** $\cos\sqrt{x+1}$
- **r** $\sin(\cos x)$
- **s** $e^{\sin x}$
- **t** $e^{\sqrt{2x-1}}$
- **u** $e^{(e^x)}$
- **v** $\ln(\sin x)$
- **w** $\ln\left(\sqrt{2x+3}\right)$
- **x** $\ln(\ln x)$
- **y** $\sin(\ln x)$
- **z** $\frac{1}{\ln x}$

2. Differentiate each of the following functions with respect to x

- **a** $e^x \sin^2 x$
- **b** $\frac{3\ln x}{2x}$
- **c** $\sqrt{x}\tan x$
- **d** $\frac{x^3+6x+11}{\cos x}$

3. Given that $x = e^y + 2y$

- **a** Find an expression for $\frac{dy}{dx}$ in terms of y
- **b** Calculate the gradient at the point $(1, 0)$

Reasoning

The curve $y = 3x^5 - 5x^3$ has three stationary points. Identify the roots, the stationary points and the points of inflection, showing your working. Hence sketch the curve.

Challenge

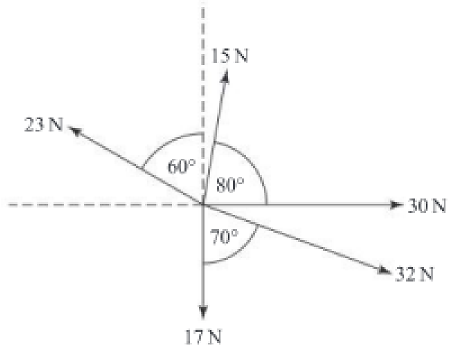
Use the reverse chain to integrate the following:

- **a** $(3x+2)^{10}$
- **b** $(5x-1)^8$
- **c** $(7x-3)^{100}$
- **d** $(3x-8)^{-8}$
- **e** $(1-3x)^9$
- **f** $(6-x)^7$
- **g** $\frac{3}{(2x-1)^5}$
- **h** $\frac{1}{(10-x)^5}$
- **i** $\sin(3-5x)$
- **j** $2\cos(4x-1)$
- **k** $\cos(2x)$
- **l** $\sec^2(4x+3)$
- **m** $3\sec^2(2x+1)$
- **n** e^{5x+2}
- **o** $\frac{7}{3x+9}$
- **p** $\frac{4}{8-x}$

Consolidation: Forces at Angles

Knowledge Check

1. Calculate



i The horizontal component, X, and the vertical component, Y, (to 3 sf) of the resultant force,

ii The magnitude of the resultant force (to 2 sf) and the angle it makes with the upward vertical (to the nearest degree).

2. A particle of mass 5 kg is pushed up a smooth slope inclined at 30° to the horizontal by a horizontal force of 40 N. Taking $g = 9.81\ \mathrm{m\,s^{-2}}$, work out

a The reaction between the particle and the plane,

b The acceleration of the particle.

3. A crate of weight 800 N lies on a rough horizontal surface. A string is attached to the crate and is pulled at 30° above the horizontal until the crate is just about to slip. At this point the tension in the string is 350 N. Resolve to find the normal reaction of the surface on the crate, R, and the coefficient of friction, μ

Reasoning

A block of mass 7 kg lies on a rough slope inclined at 35° to the horizontal. A string is attached to the block and is pulled with a force of 50 N up the slope. The coefficient of friction between the mass and the slope is 0.1. Find the acceleration of the block (to 2 sf).

Challenge

Two particles of weights 15 N and 10 N are connected by a light inextensible string which passes over a smooth pulley fixed at the end of a rough horizontal table. The 15 N weight rests on the table and the 10 N weight hangs freely below the pulley. Find the coefficient of friction between the 15 N weight and the table if the system is in limiting equilibrium.

Consolidation: Modulus Function

Knowledge Check

1. Given that $|t| = 5$ work out all possible values of $|3t + 2|$.

2.

a Sketch the graph of $y = |2x - 15|$

b Solve the equation $|2x - 15| = 3$

c Solve the inequality $|2x - 15| \leq 3$

3. In each case, the graph of $y = \mathrm{f}(x)$ is a straight line passing through the points given. Sketch the graph

i $y = \mathrm{f}(x)$ **ii** $y = |\mathrm{f}(x)|$

iii $|\mathrm{f}(x - 2)|$ **iv** $|\mathrm{f}(x)| - 2$

a $(-3, 0)$ and $(0, 6)$

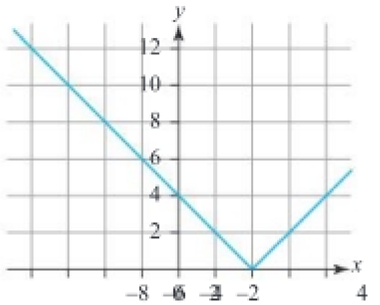
b $(-3, -2)$ and $(2, -7)$

Reasoning

Calliste attempts to answer the following question. Explain and correct her mistakes.

$\mathrm{f}(x) = 4 - x$, $x \in \mathbb{R}$. Sketch the graph of $y = |\mathrm{f}(x$

and find the values of x for which $|\mathrm{f}(x)| = \frac{1}{2}x$



$$|f(x)| = 4 + x$$

$$4 + x = \frac{1}{2}x$$

$$\frac{1}{2}x = -4$$

$$x = -8$$

Challenge

The function f is given by $\mathrm{f} : x \rightarrow |3 - 2x|$

a Sketch the graph of $y = \mathrm{f}(x)$.

b How many solutions will there be to the equation $|3 - 2x| = x$? Explain how you know.

c Solve the inequality $|3 - 2x| \geq x$, showing your working.

Consolidation: Trapezium Rule

Knowledge Check

1.

1 Use the trapezium rule with the stated number of intervals to find an estimate for these integrals. Give each estimate to two decimal places.

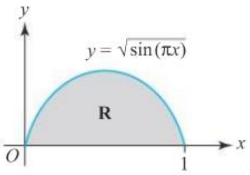
a $\int_1^5 e^{\sqrt{x}}\,dx$, 4 intervals **b** $\int_1^3 8^{\frac{1}{x}}\,dx$, 4 intervals **c** $\int_3^6 \ln\left(1+\sqrt{2x}\right)dx$, 5 intervals

2.

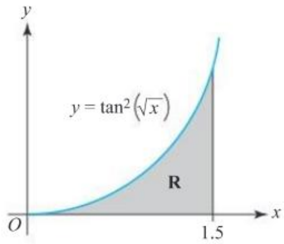
2 i Use the trapezium rule with five intervals to find an estimate for the area of the region **R** shown in each diagram. Work in radians where appropriate and give your answer to 3 dp.

ii Is your answer to part **i** an underestimate or an overestimate for the area of **R**? Why?

a



b



3.

3 The table shows some values for x and y, where $y = 3^{x^2-1}$

The values of y have been rounded to 2 dp where appropriate.

x_i	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y_i	1	1.26	1.62		2.87		5.55	7.98		17.59	27

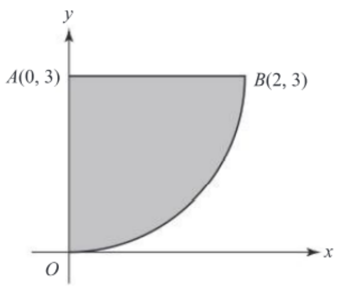
a Copy and complete the table.

b Use the trapezium rule with all of the y-values in the completed table to find an estimate for $\int_1^2 3^{x^2-1}\,dx$. Give your answer to 1 dp.

Reasoning

1 a Use the trapezium rule with five intervals to estimate the value of $\int_0^2 1-\sqrt{\cos(0.25\pi x)}\,dx$, where x is in radians. Give your answer to 2 sf.

The shaded region in the diagram shows the plan of a large field OAB where O is the origin. Point A has coordinates $(0, 3)$ and point B has coordinates $(2, 3)$. The curve OB has equation $y = 3 - 3\sqrt{\cos(0.25\pi x)}$ and all distances are measured in kilometres.



b i Use your answer to part **a** to find an estimate for the area of this field. Give your answer to 3 sf.

ii State, with a reason, whether this answer is an underestimate or an overestimate for the exact area of this field.

4 **a** **i** Use the trapezium rule with five intervals to estimate the value of $\int_0^8 2^{0.25x}\,dx$ to 1 dp.

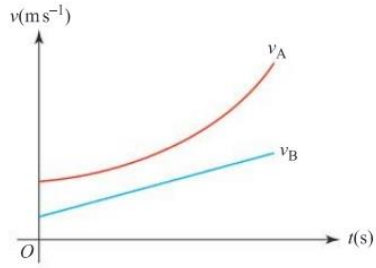
ii State, with a reason, whether your answer is an underestimate or an overestimate for this integral.

At the start of a race, runner B has a 10 metre head start on runner A. The equations $v_A = k + 2^{0.25t}$ and $v_B = 0.25t + 1$ model the speeds (in m s^{-1}) of A and B, respectively, t seconds from the start, where k is a constant. After four seconds, A is running $1\ \text{m s}^{-1}$ faster than B,

b Find the value of k

The diagram shows the speed–time graphs for runners A and B.

It takes B exactly eight seconds to complete the race.



c Determine who wins the race, justifying your answer.

Challenge

5 **a** Write down the formula for the sum of the first n terms of a geometric series with first term a and common ratio r

b Show that by using the trapezium rule with n intervals to estimate the value of $\int_0^1 2^{nx}\,dx$ the answer given is $\frac{3}{2n}\left(2^n - 1\right)$

For a particular value of n, the estimate for $\int_0^1 2^{nx}\,dx$ found using the trapezium rule with n intervals is 6144 to the nearest integer.

c **i** Use a suitable numerical method to find the value of n, justifying your answer.

ii For this n, find an estimate for the value of $\int_0^1 4^{\sqrt{nx}}\,dx$, giving your answer to 1 dp.

REVIEW: PURE PRACTICE EXAM PAPER

1. Prove, from first principles, that the derivative of $5x^3$ is $15x^2$.

(Total 4 marks)

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2. (*a*) Sketch the graph of $y = 8^x$ stating the coordinates of any points where the graph crosses the coordinate axes.

(2)

(*b*) (i) Describe fully the transformation which transforms the graph $y = 8^x$ to the graph $y = 8^{x-1}$.

(1)

(ii) Describe the transformation which transforms the graph $y = 8^{x-1}$ to the graph $y = 8^{x-1} + 5$.

(1)

(Total 4 marks)

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3. Solve algebraically, showing each step of your working, the equation

$$(8^{x-1})^2 - 18(8^{x-1}) + 32 = 0.$$

(Total 5 marks)

4.

$$g(x) = \frac{4}{x-6} + 5, x \in \mathbb{R}.$$

Sketch the graph $y = g(x)$.

Label any asymptotes and any points of intersection with the coordinate axes.

(Total 5 marks)

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5. $f(x) = x^2 - (k + 8)x + (8k + 1)$.

(*a*) Find the discriminant of $f(x)$ in terms of k giving your answer as a simplified quadratic.

(3)

(*b*) If the equation $f(x) = 0$ has two equal roots, find the possible values of k.

(2)

(*c*) Show that when $k = 8$, $f(x) > 0$ for all values of x.

(3)

(Total 8 marks)

6. The equations of two circles are $x^2 + 10x + y^2 - 12y = 3$ and $x^2 - 6x + y^2 - 2qy = 9$.

(*a*) Find the centre and radius of each circle, giving your answers in terms of q where necessary.

(6)

(*b*) Given that the distance between the centres of the circles is $\sqrt{80}$, find the two possible values of q.

(3)

(Total 9 marks)

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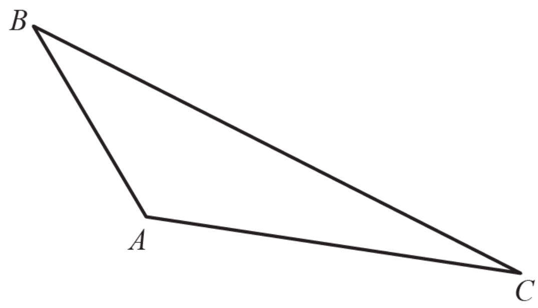
7. In $\triangle ABC$, $\overrightarrow{AB} = -3\mathbf{i} + 6\mathbf{j}$ and $\overrightarrow{AC} = 10\mathbf{i} - 2\mathbf{j}$.



(*a*) Find the size of $\angle BAC$, in degrees, to 1 decimal place.

(5)

(*b*) Find the exact value of the area of $\triangle ABC$.

(3)

(Total 8 marks)

8. The points A and B have coordinates $(3k - 4, -2)$ and $(1, k + 1)$ respectively, where k is a constant.

Given that the gradient of AB is $-\frac{3}{2}$,

(*a*) show that $k = 3$,

(2)

(*b*) find an equation of the line through A and B,

(3)

(*c*) find an equation of the perpendicular bisector of A and B. Leave your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(4)

(Total 9 marks)

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9. A stone is thrown from the top of a cliff.

The height h, in metres, of the stone above the ground level after t seconds is modelled by the function

$$h(t) = 115 + 12.25t - 4.9t^2.$$

(*a*) Give a physical interpretation of the meaning of the constant term 115 in the model.

(1)

(*b*) Write $h(t)$ in the form $A - B(t - C)^2$, where A, B and C are constants to be found.

(3)

(*c*) Using your answer to part (*b*), or otherwise, find, with justification

the time taken after the stone is thrown for it to reach ground level,

(3)

(ii) the maximum height of the stone above the ground and the time after which this maximum height is reached.

(2)

(Total 9 marks)

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10. $f(x) = x^3 + x^2 + px + q$, where p and q are constants.

Given that $f(5) = 0$ and $f(-3) = 8$,

(a) find the values of p and q,

(7)

(b) factorise $f(x)$ completely.

(5)

(Total 12 marks)

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11. (a) Given that $\int_a^{2a} (10 - 6x)\,dx = 1$, find the two possible values of a.

(6)

(b) Labelling all axes intercepts, sketch the graph of $y = 10 - 6x$ for $0 \leq x \leq 2$.

(2)

(c) With reference to the integral in part a and the sketch in part (b), explain why the larger value of a found in part (a) produces a solution for which the actual area under the graph between a and $2a$ is not equal to 1. State whether the area is greater than 1 or smaller than 1.

(2)

(Total 10 marks)

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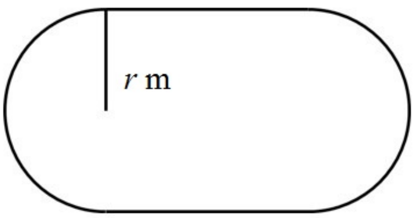
12. The diagram shows the plan of a school running track. It consists of two straight sections, which are the opposite sides of a rectangle, and two semicircular sections, each of radius r m. The length of the track is 300 m and it can be assumed to be very narrow.



(a) Show that the internal area, A m^2, is given by the formula $A = 300r - \pi r^2$.

(5)

(b) Hence find in terms of π the maximum value of the internal area.
You do not have to justify that the value is a maximum.

(6)

(Total 11 marks)

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13. A teacher asks one of her students to solve the equation $2 \cos 2x + \sqrt{3} = 0$ for $0 \leqslant x \leqslant 180°$.

The attempt is shown below.

$$2\cos 2x = -\sqrt{3}$$
$$\cos 2x = -\frac{\sqrt{3}}{2}$$
$$2x = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$$
$$2x = 150°$$
$$x = 75°$$
w or $x = 360° - 75° = 295°$ so reject as out of range.

(*a*) Identify the mistake made by the student.

(1)

(*b*) Write down the correct solutions to the equation.

(2)

(Total 3 marks)

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14. Find in exact form the unit vector in the same direction as $\mathbf{a} = 4\mathbf{i} - 7\mathbf{j}$.

(Total 3 marks)

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15. The curve with equation $y = \text{h}(x)$ passes through the point (4, 19).

Given that $\text{h}'(x) = 15x\sqrt{x} - \dfrac{40}{\sqrt{x}}$, find $\text{h}(x)$.

(Total 6 marks)

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16. Find all the solutions, in the interval $0 \leqslant x \leqslant 360°$, to the equation $8 - 7 \cos x = 6 \sin^2 x$, giving solutions to 1 decimal place where appropriate.

(Total 6 marks)

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17. (*a*) Find the first four terms, in ascending powers of x, of the binomial expansion of $(2 + px)^9$.

(4)

Given that the coefficient of the x^3 term in the expansion is -84.

(*b*) (i) Find the value of p.

(2)

(ii) Find the numerical values for the coefficients of the x and x^2 terms.

(2)

(Total 8 marks)

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18. $\log_{11}(2x - 1) = 1 - \log_{11}(x + 4)$.

Find the value of x showing detailed reasoning.

(Total 6 marks)

REVIEW: MECHANICS PRACTICE EXAM PAPER

1. A boat travels from A to B and then from B to C. The displacement from A to B is $(-28\mathbf{i} + 80\mathbf{j})$ m. The displacement from B to C is $(130\mathbf{i} + 15\mathbf{j})$ m.

(a) Find the total distance the boat travelled in moving from A to C.

(4)

(b) Find the angle the vector $\overrightarrow{AC}$ makes with the unit vector **i**.

(4)

(Total 8 marks)

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2. A racing car starts from rest at the point A and moves with constant acceleration.of 11 m s^{-2} for 8 s. The velocity it has reached after 8 s is then maintained for T s. The racing car then decelerates from this velocity to 40 m s^{-1} in a further 2 s, reaching point B.

(a) Sketch a velocity–time graph to illustrate the motion of the racing car. Include the top speed of the racing car in your sketch.

(5)

(b) Given that the distance between A and B is 1404 m, find the value of T.

(3)

(Total 8 marks)

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3. A particle of mass 6 kg is initially at rest and is then acted upon by a force $\mathbf{R} = (a\mathbf{i} + 10\mathbf{j})$ N on a bearing of 300°.

(a) Find the exact value of a.

(3)

(b) Calculate the magnitude of **R**.

(2)

(c) Work out the magnitude of the acceleration of the particle.

(2)

(d) Find the time it takes for the particle to travel a distance of 640 m.

(2)

(Total 9 marks)

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4. A particle P moves along a straight line. Initially, P is at rest at a point O on the line.

At time t s, the velocity of P is v m s^{-1}, where $v = \frac{1}{20}t(5-t)^2$, $0 \leqslant t \leqslant 8$.

Find the values of t and the corresponding values of v when the acceleration of P is instantaneously zero.

(Total 5 marks)

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5. A person runs across a field from point A to point B with a speed of 5.3 m s^{-1} and then runs back from point B to point A with a speed of 4.8 m s^{-1}.

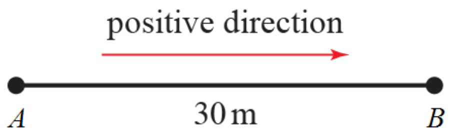

Figure 1

Taking the positive direction as shown in the diagram, state the person's

(a) velocity when travelling from A to B,

(1)

(b) velocity when travelling from B to A.

(1)

Another person runs 30 m from A in the exact opposite direction of B to a point C.

(c) State this person's displacement from A at the point C.

(1)

(Total 3 marks)

6.

A car is initially travelling with a constant velocity of 15 m s^{-1} for T s. It then decelerates at a constant rate for $\frac{T}{2}$ s, reaching a velocity of 10 m s^{-1}. It then immediately accelerates at a constant rate for $\frac{3T}{2}$ s reaching a velocity of 20 m s^{-1}.

(a) Sketch a velocity–time graph to illustrate the motion.

(3)

(b) Given that the car travels a total distance of 1312.5 m over the journey described, find the value of T.

(4)

(Total 7 marks)

7.A car of mass 1200 kg pulls a trailer of mass 400 kg along a straight horizontal road. The car and trailer are connected by a tow-rope modelled as a light inextensible rod. The engine of the car provides a constant driving force of 3200 N. The horizontal resistances of the car and the trailer are proportional to their respective masses. Given that the acceleration of the car and the trailer is 0.4 m s^{-2},

(a) find the resistance to motion on the trailer,

(4)

(b) find the tension in the tow-rope.

(3)

When the car and trailer are travelling at 25 m s^{-1} the tow-rope breaks. Assuming that the resistances to motion remain unchanged,

(c) find the distance the trailer travels before coming to a stop.

(4)

(d) State how you have used the modelling assumption that the tow-rope is inextensible.

(1)

(Total 12 marks)

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8. A particle P travels in a straight line.

At time t s, the displacement of P from a point O on the line is s m. At time t s, the acceleration of P is $(12t - 4)$ m s^{-2}. When $t = 1$, $s = 2$ and when $t = 3$, $s = 30$.

Find the displacement when $t = 2$.

(Total 8 marks)

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9. The height of a pole vaulter above the ground can be modelled using the equation $h = \frac{1}{60}\left(125x - 12x^2\right)$, where h metres is the vertical height of the pole vaulter and x metres is the horizontal distance travelled after his feet leave the ground.

(a) Find the horizontal distance travelled when the pole vaulter lands.

(3)

(b) Given that the pole vaulter is at his greatest height halfway between leaving the ground and landing, find the greatest height of the pole vaulter.

(3)

For a jump to be successful, the pole vaulter must clear a bar of height 4.9 m.

(c) Calculate the range of horizontal distances from the bar that the pole vaulter can leave the ground and have a successful jump.

(7)

(d) State the effect in this model of

(i) modelling the pole vaulter as a particle,

(1)

(ii) making air ressistance negligible.

(1)

(Total 15 marks)

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10. A box *A* of mass 0.8 kg rests on a rough horizontal table and is attached to one end of a light inextensible string. The string passes over a smooth pulley fixed at the edge of the table. The other end of the string is attached to a sphere *B* of mass 1.2 kg, which hangs freely below the pulley. The magnitude of the frictional force between *A* and the table is *F* N. The system is released from rest with the string taut. After release, *B* descends a distance of 0.9 m in 0.8 s.

Modelling *A* and *B* as particles, calculate

(a) the acceleration of *B*,

(2)

(b) the tension in the string,

(3)

(c) the value of *F*.

(3)

Sphere *B* is 0.9 m above the ground when the system is released. Given that it does not reach the pulley and the frictional force remains constant throughout,

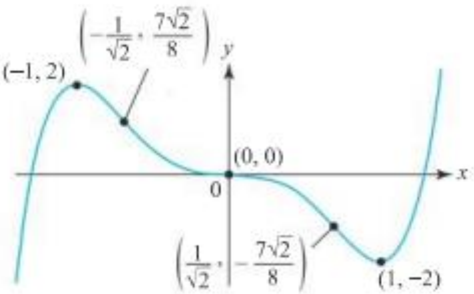
(d) find the total distance travelled by *A*.

(7)

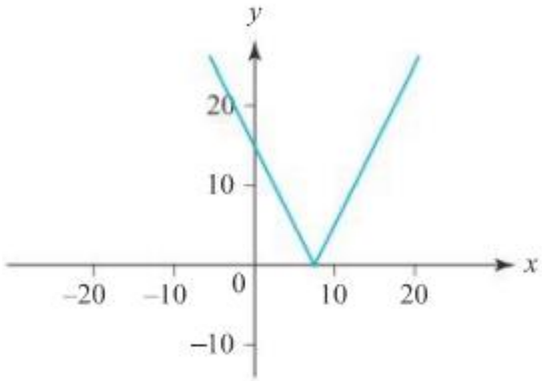
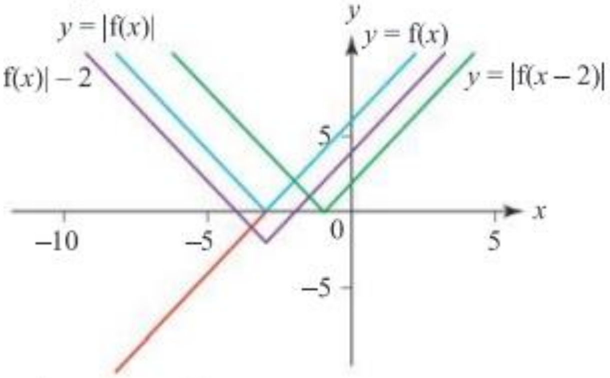
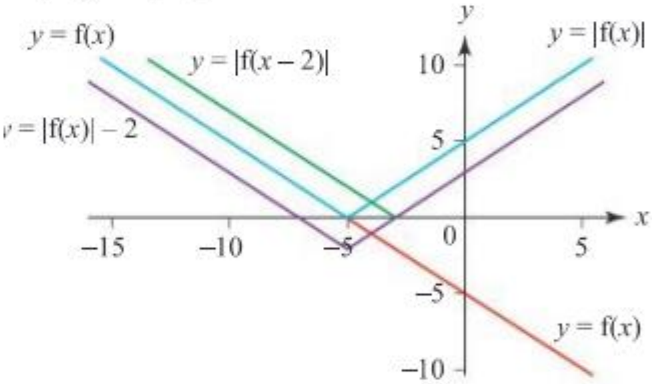
(Total 15 marks)

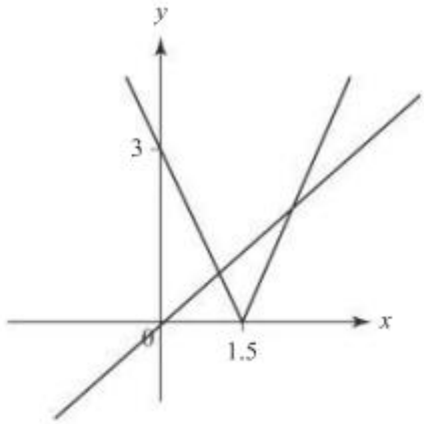
CONSOLIDATION ANSWERS

	Partial Fractions	Differentiation
1	**a** B = 4, A = 3 **b** E = 17, C = −9 and D = 4 **c** G = 3, F = 2, H = −5	**a** $15(3x+4)^4$ **b** $14(2x-1)^6$ **c** $12x(x^2+1)^5$ **d** $-6(1-2x-3x^2)^2(1+3x)$ **e** $\frac{1}{\sqrt{2x+1}}$ **f** $-\frac{5}{2\sqrt{3-5x}}$ **g** $\frac{3x}{\sqrt{3x^2+4}}$ **h** $-\frac{2}{3\sqrt[3]{(1-2x)^2}}$ **i** $\frac{2x+3}{2\sqrt{x^2+3x+4}}$ **j** $-\frac{4}{(2x+3)^3}$ **k** $\frac{3}{2}(1-3x)^{-\frac{3}{2}}$ **l** $\frac{6(1-x)}{(x^2-2x+5)^2}$ **m** $-2\cos x\sin x$ **n** $-\frac{\sin x}{2\sqrt{\cos x}}$ **o** $3\cos(3x+2)$ **p** $5\sec^2(5x-1)$ **q** $-\frac{\sin\sqrt{x+1}}{2\sqrt{x+1}}$ **r** $-\cos(\cos x)\sin x$ **s** $e^{\sin x}\cos x$ **t** $\frac{e^{\sqrt{2x-1}}}{\sqrt{2x-1}}$ **u** $e^{(e^x)}\cdot e^x$ **v** $\cot x$ **w** $\frac{1}{2x+3}$ **x** $\frac{1}{x\ln x}$ **y** $\frac{\cos(\ln x)}{x}$ **z** $-\frac{1}{x(\ln x)^2}$
2	**a** $\frac{4}{x+3}-\frac{3}{x-2}$ **b** $\frac{2}{x-1}-\frac{2}{x+1}+\frac{6}{(x+1)^2}$ **c** $\frac{1}{2x-5}-\frac{2}{x+6}-\frac{5}{(2x-1)}$	**a** $e^x\sin x(\sin x+2\cos x)$ **b** $\frac{3(1-\ln x)}{2x^2}$ **c** $\sqrt{x}\sec^2 x+\frac{\tan x}{2\sqrt{x}}$ **d** $\frac{(3x^2+6)\cos x+(x^3+6x+11)\sin x}{\cos^2 x}$
3	**a** $3-\frac{2}{x-1}+\frac{4}{x-3}$ $5+\frac{1}{(x+1)}-\frac{4}{x+5}$	**a** $\frac{1}{e^y+2}$ **b** $\frac{1}{3}$

R	Need three partial fractions in order to cope with the repeated $x+1$ term in the denominator $\frac{14}{(x-5)(x+1)^2} \equiv \frac{A}{(x-5)} + \frac{B}{(x+1)} + \frac{C}{(x+1)^2}$ $14 = A(x+1)^2 + B(x-5)(x+1) + C(x-5)$ $x=5 \Rightarrow A = \frac{7}{18}$ $x=-1 \Rightarrow C = -\frac{7}{3}$ $x^2: \ 0 = \frac{7}{18} + B \Rightarrow B = -\frac{7}{18}$ $\frac{14}{(x-5)(x+1)^2} \equiv \frac{7}{18(x-5)} - \frac{7}{18(x+1)} - \frac{7}{3(x+1)^2}$	Roots: $x = 0$ or $\pm\sqrt{\frac{5}{3}}$ Stationary points: Horizontal point of inflection at (0, 0) Maximum turning point at (–1, 2) Minimum turning point at (1, –2) Non-horizontal points of inflection, both on decreasing sections of curve at $\left(\frac{1}{\sqrt{2}}, -\frac{7\sqrt{2}}{8}\right)$, and $\left(\frac{-1}{\sqrt{2}}, \frac{7\sqrt{2}}{8}\right)$ 
C	$Q = b - ac$; $P = a$	**a** $\frac{1}{33}(3x+2)^{11} + c$ **b** $\frac{1}{45}(5x-1)^9 + c$ **c** $\frac{1}{707}(7x-3)^{101} + c$ **d** $-\frac{1}{21}(3x-8)^{-7} + c$ **e** $-\frac{1}{30}(1-3x)^{10} + c$ **f** $-\frac{1}{8}(6-x)^8 + c$ **g** $-\frac{3}{8}(2x-1)^{-4} + c$ **h** $\frac{1}{4}(10-x)^{-4} + c$ **i** $\frac{1}{5}\cos(3-5x) + c$ **j** $\frac{1}{2}\sin(4x-1) + c$ **k** $\frac{1}{2}\sin(2x) + c$ **l** $\frac{1}{4}\tan(4x+3) + c$ **m** $\frac{3}{2}\tan(2x+1) + c$ **n** $\frac{1}{5}e^{5x+2} + c$ **o** $\frac{7}{3}\ln\|3x+9\| + c$ **p** $-4\ln\|8-x\| + c$

CONSOLIDATION ANSWERS

	Forces	Modulus
1	**i** $X = 42.8\,\text{N}$ (to 3 sf) $Y = -1.67\,\text{N}$ (to 3 sf) **ii** $R = 43\,\text{N}$ (to 2 sf) $\theta = 92°$ to the nearest degree	13 or 17
2	**a** $62.5\,\text{N}$ b $2.02\,\text{m}\,\text{s}^{-2}$	**a**  **b** $x = 9$ or $x = 6$ **c** $6 \le x \le 9$
3	$R = 625\,\text{N}$ $\mu = 0.485$ (to 3 sf)	**a** $y = 2x + 6$  **b** $y = -x - 5$ 

R	$0.72\,\text{m}\,\text{s}^{-2}$ (to 2 sf)	Her mistake is that $\lvert 4-x \rvert \neq 4+x$ $\lvert f(x) \rvert = \frac{1}{2}x \Rightarrow$ $4-x=\frac{1}{2}x$ or $4-x=-\frac{1}{2}x$ $4=\frac{3}{2}x$ or $4=\frac{1}{2}x$ $x=\frac{8}{3}$ or $x=8$
C	$\mu = \frac{2}{3}$	**a**  **b** 2 as they will intersect twice – either sketch to demonstrate or explain that gradient of $y=x$ is less than gradient of $y=-(3-2x)$. **c** $x \le 1, x \ge 3$

CONSOLIDATION ANSWERS

Trapezium Rule

1

1 a 23.19 (2 dp) b 7.06 (2 dp) c 4.14 (2 dp)

2

a i 0.697 (3 dp) ii underestimate

b i 3.268 (3 dp) ii overestimate

3

3 a

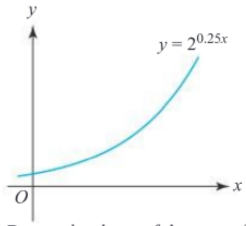
x_i	1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
y_i	1	1.26	1.62	**2.13**	2.87	**3.95**	5.55	7.98	**11.72**	17.59	27

b 6.9 (1 dp)

R

4 a i 17.4 (1 dp)

ii



Due to the shape of the curve (convex), the answer found is an overestimate for the integral.

b $k = 1$

c A's distance from start after 8 seconds $= \int_0^8 1 + 2^{0.25t}\ dt$

$\approx [t]_0^8 + 17.4$

$= 25.4\text{ m}$ (an overestimate)

B's distance from start after 8 seconds $= 10\text{ m} + \int_0^8 0.25t + 1\ dt$

$= 10\text{ m} + [0.125t^2 + t]_0^8$

$= 26\text{ m}$

Hence B wins the race

1 a 0.52 (2 sf)

b i 4.44 km² (3 sf)

ii Underestimate for the actual area of the field because 0.52110420 is an overestimate for $\int_0^2 1 - \sqrt{\cos(0.25\pi x)}\ dx$ (due to convex curve).

C

5 a $S_n = \dfrac{a(r^n - 1)}{r - 1}$

b $h = \dfrac{1-0}{n} = \dfrac{1}{n}$

x_i	0	$\frac{1}{n}$	$\frac{2}{n}$	...	$\frac{n-1}{n}$	1
y_i	1	2	2^2	....	2^{n-1}	2^n

$$\int_0^1 2^{nx}\ dx \approx \frac{1}{2}\left(\frac{1}{n}\right)[1 + 2^n + 2(2 + 2^2 + \ldots + 2^{n-1})]$$

$2 + 2^2 + \ldots + 2^{n-1}$ is the sum of the first $(n-1)$ terms of a geometric series, first term and common ratio 2

Hence $2 + 2^2 + \ldots + 2^{n-1} = \dfrac{2(2^{n-1} - 1)}{2 - 1}$

$= 2^n - 2$

$$\therefore \int_0^1 2^{nx}\ dx \approx \frac{1}{2}\left(\frac{1}{n}\right)[1 + 2^n + 2(2^n - 2)]$$

$$= \frac{1}{2n}[1 + 2^n + 2^{n+1} - 4]$$

$$= \frac{1}{2n}[2^n(1+2) - 3]$$

$$= \frac{3}{2n}(2^n - 1), \text{ as required}$$

REVIEW ANSWERS: PURE EXAM PAPER

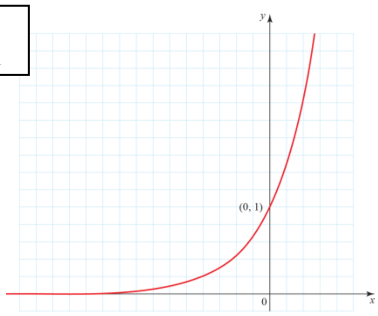
1 States or implies the formula for differentiation from first principles. $\mathrm{f}(x) = 5x^3$ $\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$	**B1**
Correctly applies the formula to the specific formula and expands and simplifies the formula. $\mathrm{f}'(x) = \lim_{h \to 0} \frac{5(x+h)^3 - 5x^3}{h}$ $\mathrm{f}'(x) = \lim_{h \to 0} \frac{5(x^3 + 3x^2h + 3xh^2 + h^3) - 5x^3}{h}$ $\mathrm{f}'(x) = \lim_{h \to 0} \frac{15x^2h + 15xh^2 + 5h^3}{h}$	**M1**
Factorises the 'h' out of the numerator and then divides by h to simplify. $\mathrm{f}'(x) = \lim_{h \to 0} \frac{h(15x^2 + 15xh + 5h^2)}{h}$ $\mathrm{f}'(x) = \lim_{h \to 0} (15x^2 + 15xh + 5h^2)$	**A1**
States that as $h \to 0$, $15x^2 + 15xh + 5h^2 \to 15x^2$ o.e. so derivative = $15x^2$ *	**A1***
	(4 marks)

NOTES: Use of δx also acceptable.

Students must show a complete proof (without wrong working) to achieve all 4 marks.

Not all steps need to be present, and additional steps are also acceptable.

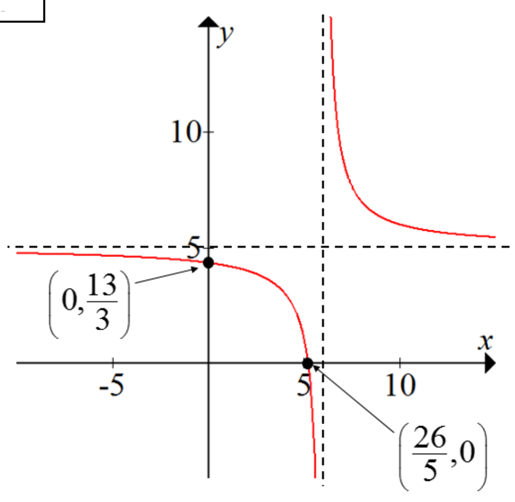
2

	Graph has correct shape and does not touch *x*-axis.	**M1**
	The point (0, 1) is given or labelled.	**A1**
		(2 marks)
Translation 1 unit right (or positive *x* direction) or by $\begin{pmatrix}1\\0\end{pmatrix}$		**B1**
Translation 5 units up (or positive *y* direction) or by $\begin{pmatrix}0\\5\end{pmatrix}$		**B1**
		(2 marks)
		Total 4 marks

3

Correctly factorises. $\left(8^{x-1}-2\right)\left(8^{x-1}-16\right)=0$ (or for example, $(y-2)(y-16)=0$)	**M1**
States that $8^{x-1}=2$, $8^{x-1}=16$ (or y = 2, y = 16).	**A1**
Makes an attempt to solve either equation (e.g. uses laws of indices. For example, $\sqrt[3]{8}=2$ or $8^{\frac{1}{3}}=2$ or $\left(\sqrt[3]{8}\right)^4=16$ or $8^{\frac{4}{3}}=16$ (or correctly takes logs of both sides).	**M1**
Solves to find $x=\frac{4}{3}$ o.e. or awrt 1.33	**A1**
Solves to find $x=\frac{7}{3}$ o.e. or awrt 2.33	**A1**
NOTE: 2nd M mark can be implied by either $x-1=\frac{1}{3}$ or $x-1=\frac{4}{3}$	**Total 5 marks**

4

	Asymptote drawn at x = 6	**B1**
	Asymptote drawn at y = 5	**B1**
	Point $\left(0,\frac{13}{3}\right)$ labelled. Condone $\frac{13}{3}$ clearly on y axis.	**B1**
	Point $\left(\frac{26}{5},0\right)$ labelled. Condone $\frac{26}{5}$ clearly on x axis.	**B1**
	Correctly shaped graph drawn in the correct quadrants formed by the asymptotes.	**B1**
		(5 marks)

5a	Statement that discriminant is $b^2 - 4ac$, and/or implied by writing $(k+8)^2 - 4\times1\times(8k+1)$	**M1**
	Attempt to simplify the expression by multiplying out the brackets. Condone sign errors and one algebraic error (but **not** missing k term from squaring brackets and must have k^2, k and constant terms). $k^2 + 8k + 8k + 64 - 32k - 4$ o.e.	**M1**
	$k^2 - 16k + 60$	**A1**
		(3 marks)
5b	Knowledge that two equal roots occur when the discriminant is zero. This can be shown by writing $b^2 - 4ac = 0$, or by writing $k^2 - 16k + 60 = 0$	**M1**
	$k = 10,\ k = 6$	**A1**
		(2 marks)
5c	Correct substitution for $k = 8$: $\mathrm{f}(x) = x^2 - 16x + 65$	**B1**
	Attempt to complete the square for their expression of f(x). $\mathrm{f}(x) = (x-8)^2 + 1$	**M1**
	<u>Statement</u> (which can be purely algebraic) that f(x) > 0, because, for example, a squared term is always greater than or equal to zero, so one more than a square term must be greater than zero or an appeal to a reasonable sketch of y = f(x).	**A1**
		(3 marks)
		Total 8 marks

NOTE:

5a: Not all steps have to be present to award full marks. For example, the second method mark can still be awarded if the answer does not include that step.

5b: Award full marks for $k = 6$, $k = 10$ seen. Award full marks for valid and complete alternative method (e.g. expanding $(x - a)^2$ comparing coefficients and solving for k).

5c: An alternative method is acceptable. For example, students could differentiate to find that the turning point of the graph of y = f(x) is at (8, 1), and then show that it is a minimum. The minimum can be shown by using properties of quadratic curves or by finding the second differential. Students must explain (a sketch will suffice) that this means that the graph lies above the x-axis and reach the appropriate conclusion.

6a Student attempts to complete the square twice for the first equation (condone sign errors). $(x+5)^2-25+(y-6)^2-36=3$ $(x+5)^2+(y-6)^2=64$	**M1**
Centre (−5, 6)	**A1**
Radius = 8	**A1**
Student attempts to complete the square twice for the second equation (condone sign errors). $(x-3)^2-9+(y-q)^2-q^2=9$ $(x-3)^2+(y-q)^2=18+q^2$	**M1**
Centre (3, q)	**A1**
Radius = $\sqrt{18+q^2}$	**A1**
	(6 marks)
6b Uses distance formula for their centres and $\sqrt{80}$. For example, $(-5-3)^2+(6-q)^2=(\sqrt{80})^2$	**M1**
Student simplifies to 3 term quadratic. For example, $q^2-12q+20=0$	**M1**
Concludes that the possible values of q are 2 and 10	**A1**
	(3 marks)
	Total 9 marks

7a	States or implies that $\overrightarrow{BC} = 13\mathbf{i} - 8\mathbf{j}$ o.e.	**M1**
	Recognises that the cosine rule is needed to solve for $\angle BAC$ by stating $a^2 = b^2 + c^2 - 2bc \times \cos A$	**M1**
	Makes correct substitutions into the cosine rule. $\left(\sqrt{233}\right)^2 = \left(\sqrt{45}\right)^2 + \left(\sqrt{104}\right)^2 - 2\left(\sqrt{45}\right)\left(\sqrt{104}\right) \times \cos A$ o.e.	**M1**
	$\cos A = -\frac{7}{\sqrt{130}}$ or awrt −0.614 (seen or implied by correct answer).	**M1**
	A = 127.9° cao	**A1**
		(5 marks)
7b	States formula for the area of a triangle. Area $= \frac{1}{2}ab\sin C$	**M1**
	Makes correct substitutions using their values from above. Area $= \frac{1}{2}\left(\sqrt{45}\right)\left(\sqrt{104}\right)\sin 127.9...^\circ$	**M1ft**
	Area = 27 (units^2)	**A1ft**
		(3 marks)
		Total 8 marks

8a 9a	Use of the gradient formula to begin attempt to find k. $\frac{k+1-(-2)}{1-(3k-4)}=-\frac{3}{2}$ or $\frac{-2-(k+1)}{3k-4-1}=-\frac{3}{2}$ (i.e. correct substitution into gradient formula and equating to $-\frac{3}{2}$).	**M1**
	$2k + 6 = -15 + 9k$ $21 = 7k$ $k = 3$* (must show sufficient, convincing and correct working).	**A1***
		(2 marks)
8b	Student identifies the coordinates of either A or B. Can be seen or implied, for example, in the subsequent step when student attempts to find the equation of the line. $A(5, -2)$ or $B(1, 4)$.	**B1**
	Correct substitution of their coordinates into $y = mx + b$ or $y - y_1 = m(x - x_1)$ o.e. to find the equation of the line. For example, $-2=\left(-\frac{3}{2}\right)(5)+b$ or $y+2=\left(-\frac{3}{2}\right)(x-5)$ or $4=\left(-\frac{3}{2}\right)(1)+b$ or $y-4=\left(-\frac{3}{2}\right)(x-1)$	**M1**
	$y=-\frac{3}{2}x+\frac{11}{2}$ or $3x+2y-11=0$	**A1**
		(3 marks)
8c	Midpoint of AB is (3, 1) seen or implied.	**B1**
	Slope of line perpendicular to AB is $\frac{2}{3}$, seen or implied.	**B1**
	Attempt to find the equation of the line (i.e. substituting their midpoint and gradient into a correct equation). For example, $1=\left(\frac{2}{3}\right)(3)+b$ or $y-1=\frac{2}{3}(x-3)$	**M1**
	$2x-3y-3=0$ or $3y-2x+3=0$. Also accept any multiple of $2x-3y-3=0$ providing a, b and c are still integers.	**A1**
		(4 marks)
		Total 9 marks

115 (m) is the height of the cliff (as this is the height of the ball when t = 0). Accept answer that states 115 (m) is the height of the cliff plus the height of the person who is ready to throw the stone or similar sensible comment.	**B1**
	(1 mark)
9b Attempt to factorise the – 4.9 out of the first two (or all) terms. $h(t) = -4.9\left(t^2 - 2.5t\right) + 115$ or $h(t) = -4.9\left(t^2 - \frac{5}{2}t\right) + 115$	**M1**
$h(t) = -4.9\left(t - 1.25\right)^2 - (-4.9)\left(1.25\right)^2 + 115$ or $h(t) = -4.9\left(t - \frac{5}{4}\right)^2 - (-4.9)\left(\frac{5}{4}\right)^2 + 115$	**M1**
$h(t) = 122.65625 - 4.9\left(t - 1.25\right)^2$ o.e. (N.B. $122.65625 = \frac{3925}{32}$) Accept the first term written to 1, 2, 3 or 4 d.p. or the full answer as shown.	**A1**
	(3 marks)
9ci Statement that the stone will reach ground level when $h(t)$ = 0, or $-4.9t^2 + 12.25t + 115 = 0$ is seen.	**M1**
Valid attempt to solve quadratic equation (could be using completed square form from part **b**, calculator or formula).	**M1**
Clearly states that t = 6.25 s (accept t = 6.3 s) is the answer, or circles that answer and crosses out the other answer, or explains that t must be positive as you cannot have a negative value for time.	**A1**
	(3 marks)
9cii hmax = awrt 123 ft A from part b.	**B1ft**
t = $\frac{5}{4}$ or t = 1.25 ft C from part b.	**B1ft**
	(2 marks)
	Total 9 marks

NOTES: c: Award 4 marks for correct final answer, with some working missing. If not correct B1 for each of A, B and C correct.

If the student answered part **b** by completing the square, award full marks for part **c**, providing their answer to their part **b** was fully correct.

10a	Makes an attempt to interpret the meaning of f(5) = 0. For example, writing 125 + 25 + 5p + q = 0	**M1**
	5p + q = −150	**A1**
	Makes an attempt to interpret the meaning of f(−3) = 8. For example writing −27 + 9 – 3p + q = 8	**M1**
	−3p + q = 26	**A1**
	Makes an attempt to solve the simultaneous equations.	**M1ft**
	Solves the simultaneous equations to find that p = −22	**A1ft**
	Substitutes their value for p to find that q = −40	**A1ft**
		(7 marks)
10b	Draws the conclusion that (x – 5) must be a factor.	**M1**
	Either makes an attempt at long division by setting up the long division, or makes an attempt to find the remaining factors by matching coefficients. For example, stating: $\left(x-5\right)\left(ax^2+bx+c\right)=x^3+x^2-22x-40$ (ft their −22 or −40)	**M1ft**
	For the long division, correctly finds the the first two coefficients. For the matching coefficients method, correctly deduces that a = 1 and c = 8	**A1**
	For the long division, correctly completes all steps in the division. For the matching coefficients method, correctly deduces that b = 6	**A1**
	States a fully correct, fully factorised final answer: (x – 5)(x + 4)(x + 2)	**A1**
		(5 marks)
		Total 12 marks

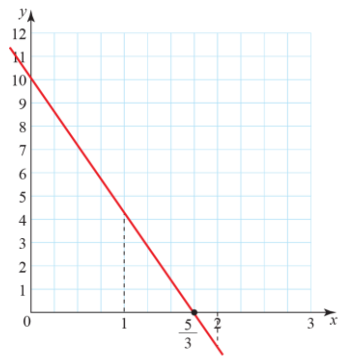
NOTES: 10a: Award ft through marks for correct attempt/answers to solving their simultaneous equations.

10b: Other algebraic methods can be used to factorise: x – 5 is a factor (M1)

$x^3-x^2-22x-40=x^2(x-5)+6x(x-5)+8(x-5)$ by balancing (M1)

$=(x^2+6x+8)(x-5)$ by factorising (M1)

$=(x+4)(x+2)(x-5)$ by factorising (A1 A1) (i.e. A1 for each factor other than (x – 5))

11aa Makes an attempt to find $\int(10-6x)\mathrm{d}x$ Raising x powers by 1 would constitute an attempt.		**M1**
Shows a fully correct integral with limits. $\left[10x-3x^2\right]_a^{2a}=1$		**A1**
Makes an attempt to substitute the limits into their expression. For example, $\left(10(2a)-3(2a)^2\right)-\left(10(a)-3(a)^2\right)$ or $\left(20a-12a^2\right)-\left(10a-3a^2\right)$ is seen.		**M1ft**
Rearranges to a 3-term quadratic equation (with = 0). $9a^2-10a+1=0$		**M1ft**
Correctly factorises the LHS: $(9a-1)(a-1)=0$ or uses a valid method for solving a quadratic equation (can be implied by correct answers).		**M1ft**
States the two fully correct answers $a=\frac{1}{9}$ or $a=1$ For the first solution accept awrt 0.111		**A1**
		(6 marks)
11ba **Figure 1**  	Straight line sloping downwards with positive x and y intercepts. Ignore portions of graph outside $0\leqslant x\leqslant 2$	**M1**
	Fully correct sketch with points (0, 10), and ($\frac{5}{3}$, 0) labelled. Ignore portions of graph outside $0\leqslant x\leqslant 2$	**A1**
		(2 marks)
11ca	Statements to the effect that the (definite) integral will only equal the area (1) if the function is above the x-axis (between the limits) AND when $a=1$, $2a=2$, so part of the area will be above the x-axis and part will be below the x-axis.	**B1**
	Greater than 1.	**B1**
		(2 marks)
		Total 10 marks

12aa tates that the perimeter of the track is $2\pi r+2x=300$ he choice of the variable *x* is not important, but there should be a variable other than *r*.	**M1**
Correctly solves for *x*. Award method mark if this is seen in a subsequent step. $x=\dfrac{300-2\pi r}{2}=150-\pi r$	**A1**
States that the area of the shape is $A=\pi r^2+2rx$	**B1**
Attempts to simplify this by substituting their expression for *x*. $A=\pi r^2+2r\left(150-\pi r\right)$ $A=\pi r^2+300r-2\pi r^2$	**M1**
States that the area is $A=300r-\pi r^2$ *	**A1***
	(5 marks)
Attempts to differentiate *A* with respect to *r*	**M1**
12ba nds $\dfrac{dA}{dr}=300-2\pi r$	**A1**
Shows or implies that a maximum value will occur when $300-2\pi r=0$	**M1**
Solves the equation for *r*, stating $r=\dfrac{150}{\pi}$	**A1**
Attempts to substitute for *r* in $A=300r-\pi r^2$, for example writing $A=300\left(\dfrac{150}{\pi}\right)-\pi\left(\dfrac{150}{\pi}\right)^2$	**M1**
Solves for *A*, stating $A=\dfrac{22\,500}{\pi}$	**A1**
	(6 marks)
	Total **11 marks**

NOTES: 12b: Ignore any attempts at deriving second derivative and related calculations.

13 Any reasonable explanation. For example, the student did not correctly find all values of $2x$ which satisfy $\cos 2x = -\frac{\sqrt{3}}{2}$. Student should have subtracted 150° from 360° first, and then divided by 2. N.B. If insufficient detail is given but location of error is correct then mark can be awarded from working in part (b).	**B1**
	(1 mark)
x = 75°	**B1**
x = 105°	**B1**
	(2 marks)
	Total 3 marks

NOTE: 13a: Award the mark for a different explanation that is mathematically correct, provided that the explanation is clear and not ambiguous.

14 Makes an attempt to use Pythagoras' theorem to find $\lvert\mathbf{a}\rvert$. For example, $\sqrt{(4)^2 + (-7)^2}$ seen.	**M1**
$\sqrt{65}$	**A1**
Displays the correct final answer. $\frac{1}{\sqrt{65}}(4\mathbf{i} - 7\mathbf{j})$	**A1**
	(3 marks)

15 Uses laws of indices correcty at least once anywhere in solution (e.g. $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$ or $\sqrt{x} = x^{\frac{1}{2}}$ or $x\sqrt{x} = x^{\frac{3}{2}}$ seen or implied).	**B1**
Makes an attempt at integrating $\mathrm{h}'(x) = 15x^{\frac{3}{2}} - 40x^{-\frac{1}{2}}$ Raising at least one x power by 1 would constitute an attempt.	**M1**
Fully correct integration. $6x^{\frac{5}{2}} - 80x^{\frac{1}{2}}$ (no need for $+C$ here).	**A1**
Makes an attenpt to substitute (4, 19) into the integrated expression. For example, $19 = 6 \times 4^{\frac{5}{2}} - 80 \times 4^{\frac{1}{2}} + C$ is seen.	**M1**
Finds the correct value of C. $C = -13$	**A1**
States fully correct final answer $\mathrm{h}(x) = 6x^{\frac{5}{2}} - 80\sqrt{x} - 13$ or any equivalent form.	**A1**
	(6 marks)

NOTES: Award all 6 marks for a fully correct final answer, even if some working is missing.

16 States $\sin^2 x + \cos^2 x = 1$ or implies this by making a substitution. $8 - 7\cos x = 6\left(1 - \cos^2 x\right)$	**M1**
Simplifies the equation to form a quadratic in cos *x*. $6\cos^2 x - 7\cos x + 2 = 0$	**M1**
Correctly factorises this equation. $(3\cos x - 2)(2\cos x - 1) = 0$ or uses equivalent method for solving quadratic (can be implied by correct solutions).	**M1**
Correct solution. $\cos x = \frac{2}{3}$ or $\frac{1}{2}$	**A1**
Finds one correct solution for *x*. (48.2°,60°, 311.8° or 300°).	**A1**
Finds all other solutions to the equation.	**A1**
	(6 marks)

17	States or implies the expansion of a binomial expression to the 9th power, up to and including the x^3 term. $(a+b)^9 = {}^9C_0a^9 + {}^9C_1a^8b + {}^9C_2a^7b^2 + {}^9C_3a^6b^3 + \ldots$ or $(a+b)^9 = a^9 + 9a^8b + 36a^7b^2 + 84a^6b^3 + \ldots$	**M1**
	Correctly substitutes 2 and px into the formula. $(2+px)^9 = 2^9 + 9\times 2^8 \times px + 36\times 2^7 \times (px)^2 + 84\times 2^6 \times (px)^3 + \ldots$	**M1**
	Makes an attempt to simplify the expression (at least one power of 2 calculated and one bracket expanded correctly). $(2+px)^9 = 512 + 9\times 256\times px + 36\times 128\times p^2x^2 + 84\times 64\times p^3x^3 + \ldots$	**M1dep**
	States a fully correct answer: $(2+px)^9 = 512 + 2304px + 4608p^2x^2 + 5376p^3x^3 + \ldots$	**A1**
		(4 marks)
bi	States that $5376p^3 = -84$	**M1ft**
	Correctly solves for p: $p^3 = -\frac{1}{64}$ so $p = -\frac{1}{4}$	**A1ft**
bii	Correctly find the coefficient of the x term: $2304\ (-\frac{1}{4}) = -576$	**B1ft**
	Correctly find the coefficient of the x^2 term: $4608\ (-\frac{1}{4})^2 = 288$	**B1ft**
		(4 marks)
		Total: 8 marks

NOTES: ft marks – pursues a correct method and obtains a correct answer or answers from their 5376 from part **a**.

18	Uses appropriate law of logarithms to write $\log_{11}(2x-1)(x+4)=1$	**M1**
	Inverse $\log_{11}$ (or 11 to the) both sides. $(2x-1)(x+4)=11$	**M1**
	Derives a 3 term quadratic equation. $2x^2+7x-15=0$	**M1**
	Correctly factorises $(2x-3)(x+5)=0$ or uses appropriate technique to solve their quadratic.	**M1**
	Solves to find $x=\frac{3}{2}$	**A1**
	Understands that $x \neq -5$ stating that this solution would require taking the log of a negative number, which is not possible.	**B1**
		Total: 6 marks

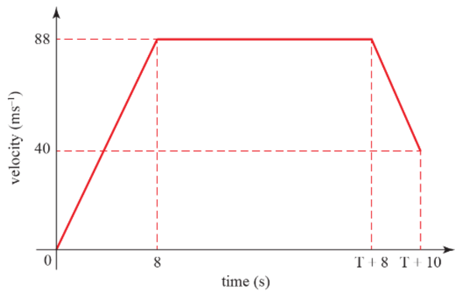
	Total: 6 marks

REVIEW ANSWERS: MECHANICS PRACTICE EXAM PAPER

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
1a	Makes an attempt to find the distance from *A* to *B*. For example, $\sqrt{(-28)^2+(80)^2}$ is seen.	**M1**	3.1b	4th Find the magnitude and direction of a vector quantity.
	Makes an attempt to find the distance from *B* to *C*. For example, $\sqrt{(130)^2+(15)^2}$ is seen.	**M1**	3.1b	
	Demonstrates an understanding that these two values need to be added. For example, 84.75… + 130.86… is seen.	**M1**	1.1b	
	215.62… (m) Accept anything which rounds to 216 (m)	**A1**	1.1b	
		(4)		
1b	States that $\overrightarrow{AC} = 102\mathbf{i} + 95\mathbf{j}$ (m) Award one point for each value.	**B2**	3.1b	4th Find the magnitude and direction of a vector quantity.
	States or implies that $\tan\theta = \frac{95}{102}$	**M1**	1.1b	
	Finds $\theta = 42.96...°$ Accept awrt 43.0°	**A1**	1.1b	
		(4)		
				(8 marks)

Notes

<table>
<tr><th>Q</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th><th>Pearson Progression Step and Progress descriptor</th></tr>
<tr><td rowspan="6">2a</td><td colspan="2">Velocity = acceleration × time seen or implied.</td><td>M1</td><td>3.1b</td><td rowspan="5">4th

Use and interpret graphs of velocity against time.</td></tr>
<tr><td colspan="2">Velocity = 11 × 8 = 88 m s^{-1}</td><td>A1</td><td>1.1b</td></tr>
<tr><td rowspan="3">Figure 2

</td><td>General shape of the graph is correct. i.e. positive gradient, followed by horizontal line, followed by negative gradient not returning to zero.</td><td>M1</td><td>3.3</td></tr>
<tr><td>Vertical axis labelled correctly.</td><td>A1</td><td>1.1b</td></tr>
<tr><td>Horizontal axis labelled correctly.</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(5)</td><td></td><td></td></tr>
<tr><td rowspan="4">2b</td><td colspan="2">Makes an attempt to find the area of the trapezoidal section. For example, $2\times\frac{1}{2}(88+40)$ is seen.</td><td>M1</td><td>1.1b</td><td rowspan="3">4th

Calculate and interpret areas under velocity–time graphs.</td></tr>
<tr><td colspan="2">Demonstrates an understanding that the three areas must total 1404. For example, $\frac{1}{2}(8\times 88)+88T+2\times\frac{1}{2}(88+40)=1404$ or $352+88T+128=1404$ is seen.</td><td>M1</td><td>2.1</td></tr>
<tr><td colspan="2">Correctly solves to find T = 10.5 (s).</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td><td></td></tr>
<tr><td colspan="6" align="right">(8 marks)</td></tr>
</table>

Notes

2a

Accept the horizontal axis labelled with the correct intervals.

2b

Award full marks for correct final answer, even if some work is missing.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
3a	Either states that $\tan 30 = \frac{10}{a}$ or $\tan 60 = \frac{a}{10}$	M1	1.1b	5th Use Newton's second law to model motion in two directions.
	Correctly find $a = 10\sqrt{3}$	M1	1.1b	
	Interprets a in the context of the question, stating $a = -10\sqrt{3}$	A1	3.2	
		(3)		
3b	States that the magnitude of $\mathbf{R} = \sqrt{\left(-10\sqrt{3}\right)^2 + (10)^2}$	M1	1.1b	5th Use Newton's second law to model motion in two directions.
	States $R = 20$ (N).	A1 ft	1.1b	
		(2)		
3c	States $F = ma$ or implies use of $F = ma$. For example $20 = 6 \times a$ is seen.	M1	3.3	5th Use Newton's second law to model motion in two directions.
	Correctly finds $a = \frac{10}{3}$ m s^{-2}.	A1 ft	1.1b	
		(2)		
3d	States that $s = ut + \frac{1}{2}at^2$ or implies it use by writing $640 = (0)t + \frac{1}{2} \times \frac{10}{3} \times t^2$	M1	3.1b	5th Use Newton's second law to model motion in two directions.
	Solves to find $t = 8\sqrt{6}$ (s). Accept awrt 19.6 (s).	A1 ft	1.1b	
		(2)		
				(9 marks)

Notes

3b

Award ft marks for a correct answer using their value from part **a** for the **i** component of the force.

3c

Award ft marks for a correct answer using their value from part **b** for the resultant force.

3d

Award ft marks for a correct answer using their value from part **c** for the acceleration.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
4	$t = 5$, $v = 0$	**B1**	1.1b	6th Uses differentiation to solve problems in kinematics.
	Expands brackets and attempts differentiation. Reducing any power by one is sufficient evidence of differentiation.	**M1**	3.1b	
	Solves $25 - 20t + 3t^2 = 0$ to find $t = \frac{5}{3}$. The expression can be factorised, or the quadratic formula can be used. $t = 5$ does not have to be seen to award the mark.	**A1**	1.1b	
	Makes an attempt to substitute $t = \frac{5}{3}$ into $v = \frac{1}{20}t(5-t)^2$. For example, $v = \left(\frac{1}{20}\right)\left(\frac{5}{3}\right)\left(\frac{10}{3}\right)^2$ is seen.	**M1**	2.2a	
	Correctly finds $v = \frac{25}{27}$ or 0.92… (m s^{-1}). Accept awrt 0.9 (m s^{-1}).	**A1 ft**	1.1b	
		(5)		
				(7 marks)

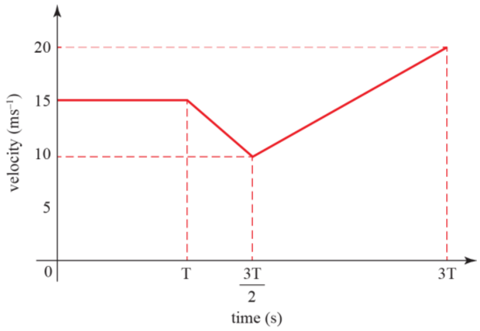
Notes

4

Award the final method mark and the final accuracy mark for a correct substitution using their value for *t*.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
5a	States correct answer: 5.3 ($m\,s^{-1}$)	**B1**	2.2a	4th Understand the difference between a scalar and a vector.
		(1)		
5b	States correct answer: −4.8 ($m\,s^{-1}$)	**B1**	2.2a	4th Understand the difference between a scalar and a vector.
		(1)		
5c	States correct answer: −30 (m)	**B1**	2.2a	4th Understand the difference between a scalar and a vector.
		(1)		
				(3 marks)

Notes

<table>
<tr><th>Q</th><th colspan="2">Scheme</th><th>Marks</th><th>AOs</th><th>Pearson Progression Step and Progress descriptor</th></tr>
<tr><td rowspan="4">6a</td><td rowspan="3">Figure 1

</td><td>General shape of the graph is correct. i.e. horizontal line, followed by negative gradient, followed by a positive gradient.</td><td>M1</td><td>3.3</td><td rowspan="3">4th
Use and interpret graphs of velocity against time.</td></tr>
<tr><td>Vertical axis labelled correctly.</td><td>A1</td><td>1.1b</td></tr>
<tr><td>Horizontal axis labelled correctly.</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(3)</td><td></td><td></td></tr>
<tr><td rowspan="5">6b</td><td colspan="2">Makes an attempt to find the area of trapezoidal section where the car is decelerating. For example, $\frac{T}{4}(15+10)$ is seen.</td><td>M1</td><td>1.1b</td><td rowspan="4">4th
Calculate and interpret areas under velocity–time graphs.</td></tr>
<tr><td colspan="2">Makes an attempt to find the area of the trapezoidal section where the car is accelerating. For example, $\frac{3T}{4}(10+20)$ is seen.</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">States that $15T+\frac{25T}{4}+\frac{90T}{4}=1312.5$</td><td>M1</td><td>1.1b</td></tr>
<tr><td colspan="2">Solves to find the value of T: $T=30$ (s).</td><td>A1</td><td>1.1b</td></tr>
<tr><td colspan="2"></td><td>(4)</td><td></td><td></td></tr>
<tr><td colspan="6" align="right">(7 marks)</td></tr>
</table>

Notes

6a

Accept the horizontal axis labelled with the correct intervals.

6b

Award full marks for correct final answer, even if some work is missing.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
7a	States, or implies in a subsequent step, that the resistances to motion will total 1600k (N). (Any variable is acceptable.)	M1	3.1b	4th Solve problems of connected particles in one dimension.
	Uses $F = ma$ to write $3200 - 1600k = 1600(0.4)$	M1	3.3	
	Solves the equation to find $k = 1.6$	A1	1.1b	
	Finds the resistance forces acting on the trailer: $R_{\text{trailer}} = 400 \times 1.6 = 640$ (N).	A1	1.1b	
		(4)		
7b	Demonstrates an understanding that the resultant force for the trailer is $T - 640$, or for the car is $3200 - 1920 - T$	M1	3.1b	4th Solve problems of connected particles in one dimension.
	Either states $T - 640 = 400(0.4)$ using the trailer or states $3200 - 1920 - T = 1200(0.4)$ using the car.	M1	3.3	
	Correctly finds $T = 800$ (N).	A1 ft	1.1b	
		(3)		
7c	Uses $F = ma$ to write $-640 = 400a$	M1	3.3	4th Solve problems of connected particles in one dimension.
	Correctly solves to find $a = -1.6\ \text{m s}^{-2}$	A1 ft	1.1b	
	Uses $v^2 = u^2 + 2as$ to write $0 = 25^2 + 2(-1.6)s$	M1	3.1b	
	Correctly solves to find $s = 195.31\ldots$ (m). Accept awrt 195 (m).	A1 ft	1.1b	
		(4)		
7d	States 'the acceleration of the car will be equal to the acceleration of the trailer' or states 'the car and the trailer will move as one'.	B1	3.5	4th Solve problems of connected particles in one dimension.
		(1)		
				(12 marks)

Notes

7b

Award ft marks for a correct answer using their value from part **a** for the resistance acting on the trailer.

7c

Award ft marks for a correct answer using their value from part **a** for the resistance acting on the trailer and from part **b** for tension.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
8	Integrates $a = 12t - 4$ to obtain $v = 6t^2 - 4t + A$ Any constant is acceptable.	**M1**	3.1b	6th Uses integration to solve problems in kinematics.
	Integrates $v = 6t^2 - 4t + A$ to obtain $s = 2t^3 - 2t^2 + At + B$. Any constant are acceptable.	**M1**	3.1b	
	Makes an attempt to form a pair of simultaneous equations by separately substituting (1, 2) and (3, 30) into the equation. For example: $2 = 2 - 2 + A + B$ and $30 = 54 - 18 + 3A + B$ are seen.	**M1**	3.1b	
	Simplifies to obtain a correctly pair of simultaneous equations: $A + B = 2$ and $3A + B = -6$ are seen.	**M1**	1.1b	
	Solves to find $A = -4$	**A1**	1.1b	
	Solves to find $B = 6$	**A1**	1.1b	
	Attempts to make a substitution of $t = 2$ into $s = 2t^3 - 2t^2 - 4t + 6$ For example, $s = 2(2)^3 - 2(2)^2 - 4(2) + 6$ is seen.	**M1**	1.1b	
	Correctly finds $s = 6$ (m).	**A1 ft**	1.1b	
		(8)		
				(8 marks)

Notes

8

Award the final method mark and the final accuracy mark for a correct substitution using their values for *A* and *B*.

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
9a	Understands that the pole vaulter will land when h = 0 or writes $\frac{1}{60}\left(125x-12x^2\right)=0$	**M1**	3.1b	3rd Understand how mechanics problems can be modelled mathematically.
	Correctly factorises to get $x\left(125-12x\right)=0$ o.e.	**M1**	1.1b	
	Solves to get $x=\frac{125}{12}=10.41...$ (m) Accept awrt 10.4 (m)	**A1**	1.1b	
		(3)		
9b	States that the greatest height will occur when x = 5.20…(m)	**M1**	3.1b	3rd Understand how mechanics problems can be modelled mathematically.
	Makes an attempt to substitute x = 5.20…into the equation for h. For example, $h=\frac{1}{60}\left(125\left(5.20...\right)-12\left(5.20...\right)^2\right)$ seen.	**M1**	1.1b	
	h = 5.42…(m) Accept awrt 5.4 (m)	**A1 ft**	1.1b	
		(3)		
9c	States h = 4.9 or states that $\frac{1}{60}\left(125x-12x^2\right)=4.9$	**M1**	3.1b	3rd Understand how mechanics problems can be modelled mathematically.
	Simplifies this to reach $12x^2-125x+294=0$ o.e.	**M1**	1.1b	
	Realises that the quadratic formula is needed to solve the quadratic. For example $a=12, b=-125, c=294$ seen, or makes attempt to use the formula: $x=\frac{125\pm\sqrt{\left(-125\right)^2-4\left(12\right)\left(294\right)}}{2\left(12\right)}$	**M1**	1.1b	
	Simplifies the b^2-4ac part to get 1513 or shows $x=\frac{125\pm\sqrt{1513}}{24}$	**M1**	1.1b	
	x = 6.82…(m) Accept awrt 6.8 (m)	**A1**	1.1b	

	$x = 3.58\ldots$ (m) Accept awrt 3.6 (m)	**A1**	1.1b	
	The pole vaulter can leave the ground between 3.6 m and 6.8 m from the bar.	**B1**	3.2a	
		(7)		
9di	Allows the person to be treated as a single mass and allows the effects of rotational forces to be ignored.	**B1**	3.4	3rd Understand assumptions common in mathematical modelling.
		(1)		
9dii	The effects of air resistance can be ignored.	**B1**	3.4	3rd Understand assumptions common in mathematical modelling.
		(1)		
				(15 marks)

Notes

9b

For the first method mark, accept their answer to part **a** divided by 2. Continue to award marks for a correct answer using their initial incorrect value.

9c

Accept $3.9 \leqslant x \leqslant 9.8$

Q	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
10a	Correctly uses $s = ut + \frac{1}{2}at^2$ to write $0.9 = (0)t + \frac{1}{2} \times a \times (0.8)^2$	**M1**	3.1b	5th Solve problems of connected particles using pulleys.
	Correctly finds $a = \frac{45}{16}$ (m s^{-2}) or 2.8125 (m s^{-2}). Accept awrt 2.8 (m s^{-2}).	**A1**	1.1b	
		(2)		
10b	Demonstrates an understanding that the resultant force acting on sphere B is $1.2g - T$.	**M1**	3.1b	5th Solve problems of connected particles using pulleys.
	Uses $F = ma$ to write $1.2g - T = 1.2\left(\frac{45}{16}\right)$	**M1**	3.3	
	Correctly solves to find $T = \frac{1677}{200}$ (N) or 8.385 (N). Accept 8.4 (N).	**A1 ft**	1.1b	
		(3)		
10c	Demonstrates an understanding that the resultant force acting on box A is $T - F$.	**M1**	3.1b	5th Solve problems of connected particles using pulleys.
	Uses $F = ma$ to write $\frac{1677}{200} - F = 0.8\left(\frac{45}{16}\right)$	**M1**	3.3	
	Correctly solves to find $F = \frac{1227}{200}$ (N) or 6.135 (N). Accept 6.1 (N).	**A1ft**	1.1b	
		(3)		
10d	Uses $v = u + at$ to write $v = 0 + \frac{45}{16} \times 0.8$	**M1**	3.1b	5th Solve problems of connected particles using pulleys.
	Solves to find $v = \frac{9}{4}$ or 2.25 m s^{-1}.	**A1 ft**	1.1b	
	Uses $F = ma$ to write $-F = 0.8a$ or $-\frac{1227}{200} = 0.8a$	**M1**	3.1b	
	Solves to find $a = -\frac{1227}{160}$ m s^{-2} or 7.66…(m s^{-2}).	**A1 ft**	1.1b	

	Uses $v^2 = u^2 + 2as$ to write $0 = \left(\frac{9}{4}\right)^2 + 2\left(-\frac{1227}{160}\right)s$	**M1**	2.2a	
	Solves to find $s = \frac{135}{409}$ (m) or 0.33… (m). Accept awrt 0.33 (m).	**A1 ft**	1.1b	
	States that the total distance travelled will be 1.23 m (0.9 + 0.33).	**B1 ft**	3.2	
		(7)		
				(15 marks)

Notes

10b

Award ft marks for a correct answer using their value from part **a** for acceleration.

10c

Award ft marks for a correct answer using their values from part **a** for acceleration and part **b** for tension.

10d

Award ft marks for a correct answer using their values from parts **a**, **b** and **c**.

Optional Extension

Have a look at the “Risps” using:

http://www.s253053503.websitehome.co.uk/risps/risplist.html

Try to expand your mathematical knowledge by trying at least **5** of them.

Here are some recommended ones that you might find interesting:

Risp 5: Tangent through the Origin

Risp 8: Arithmetic Simultaneous Equations

Risp 9: A Circle Property

Risp 16: Never Positive

Risp 19: When does fg equal gf?

Risp 19: Extending the Binomial Theorem

Risp 24: The 3-Fact Triangles

Risp 37: Parabolic Clues