New College Bradford

A-Level Further Mathematics

Y12-13

Summer Independent Learning

	Content	Answers
Compulsory		
Consolidation	Confidence Intervals Contingency Tables Collisions	Confidence Intervals Contingency Tables Collisions
Preview	Year 13 Content	
Optional		
Extension	Optional Extension	

Please bring all your SIL with you to your first lesson of Y13 Further Maths ☺

Confidence Intervals

Question 1

(a) A k% confidence interval was constructed for the population mean μ of a normal distribution with a known variance of 36, using a sample of size 25.

The calculated interval was (33.926, 37.874).

Find k.

[4 marks]

(b) The sample size, from the same population, is increased, leaving the value of k unchanged.

State the likely effect on the confidence interval.

[1 mark]

Question 2

The manufacturer of 'Nutty Nutty' peanut butter claims that the mean saturated fat content of a 454 gram jar is 58.4 grams.

The saturated fat content of any jar of 'Nutty Nutty' can be assumed to be normally distributed with a standard deviation of 12.6 grams.

A random sample of 16 jars was selected and the saturated fat content, x grams, of each jar was measured. It was found that $\Sigma x = 963.2$

(a) Find a 95% confidence interval for the mean saturated fat content of a 454 gram jar of 'Nutty Nutty' peanut butter.

[4 marks]

(b) Comment on the manufacturer's claim that the mean saturated fat content of a jar of 'Nutty Nutty' peanut butter is 58.4 grams.

[1 mark]

Contingency Tables

Question 1

A dairy industry researcher, Robyn, decided to investigate the milk yield, classified as low, medium or high, obtained from four different breeds of cow, A, B, C and D.

The milk yield of a sample of 105 cows was monitored and the results are summarised in contingency **Table 1**.

Table 1		**Yield**			
		Low	**Medium**	**High**	**Total**
Breed	**A**	4	5	12	**21**
	B	10	6	4	**20**
	C	8	17	7	**32**
	D	5	20	7	**32**
	Total	**27**	**48**	**30**	**105**

The sample of cows may be regarded as random.

Robyn decides to carry out a χ^2-test for association between milk yield and breed using the information given in **Table 1**.

(a) Contingency **Table 2** gives some of the expected frequencies for this test.

Complete **Table 2** with the missing expected values.

[2 marks]

Table 2		**Yield**		
		Low	**Medium**	**High**
Breed	**A**			6
	B	5.14	9.14	5.71
	C			
	D	8.23	14.63	9.14

(b) (i) For Robyn's test, the test statistic $\sum \frac{(O-E)^2}{E}$ = 19.4 correct to three significant figures.

Use this information to carry out Robyn's test, using the 1% level of significance.

[5 marks]

(b) (ii) By considering the observed frequencies given in **Table 1** with the expected frequencies in **Table 2**, interpret, in context, the association, if any, between milk yield and breed.

[2 marks]

Question 2

Wendy is the owner of a driving school.

She is interested in seeing whether there is a link between the number of lessons taken by the school's students before attempting their practical driving test, and the outcome of the test.

The results of some of her students who took their test in 2016 are summarised in **Table 1** below.

Table 1

Outcome of test	**Number of lessons taken**			
	0–9	10–19	20+	Total
Pass	42	50	31	123
Fail	17	24	36	77
Total	59	74	67	200

(a) State one assumption about the sample data required for the validity of a χ^2 test.

[1 mark]

(b) Wendy decides to carry out a χ^2 test for association between the outcomes of the practical driving test and the number of lessons taken.

Table 2, below, gives two of the expected frequencies to one decimal place.

Table 2

Outcome of test	**Number of lessons taken**			
	0–9	10–19	20+	Total
Pass			**41.2**	123
Fail			**25.8**	77
Total	59	74	67	200

(b) (i) Complete **Table 2** with the missing expected frequencies, giving your answers to 1 decimal place.

[2 marks]

(b) (ii) Wendy uses the data in **Tables 1** and **2** to carry out her χ^2 test and obtains a test statistic of 10.0.

Complete the test at the 1% significance level.

[5 marks]

(b) (iii) Wendy claims that if all of the observed frequencies in **Table 1** were doubled, the final value of the test statistic would remained unchanged.

She reasons that the value of the test statistic is independent of scaling.

Is Wendy correct?

Justify your answer.

[1 mark]

Question 3

Celia is the catering manager of a chain of hotels. She is investigating whether there is an association between the age of a guest and the breakfast drink they choose.

Celia selects a random sample of 500 guests and the results are summarised in the table shown below.

		Breakfast Drink			
		Tea	**Coffee**	**Fruit Juice or Water**	**Totals**
Age of Guest (years)	**Age < 25**	16	52	82	150
	25 ≤ Age < 50	44	115	41	200
	Age ≥ 50	90	43	17	150
	Totals	150	210	140	500

(a) Carry out a suitable test to determine if there is an association between the age of a guest and the breakfast drink they choose.

Fully justify your answer.

[8 marks]

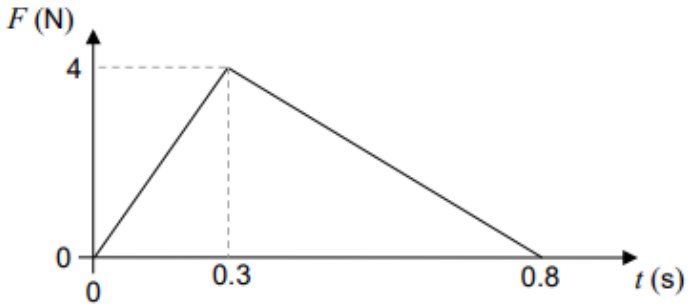
(b) Celia suggests that the test would be improved by splitting the under 25 age category into a category for children aged up to 16 and then another for those aged 16 to 25. It is known that in the sample of 500 people only 15 were aged 16 to 25. Comment on Celia's suggestion.

[1 mark]

Collisions

Question 1

The graph shows how a force, F, varies with time during a period of 0.8 seconds.



Find the magnitude of the impulse of F during the 0.8 seconds.

Circle your answer.

[1 mark]

1.0 Ns 1.6 Ns 2.2 Ns 3.2 Ns

Question 2

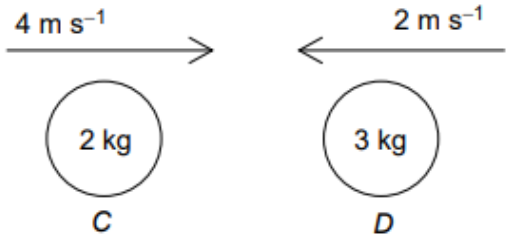
Two small smooth discs, C and D, have equal radii and masses of 2 kg and 3 kg respectively.

The discs are sliding on a smooth horizontal surface towards each other and collide directly.

Disc C has speed 4 m s^{-1} and disc D has speed 2 m s^{-1} as they collide.

The coefficient of restitution between C and D is 0.6

The diagram shows the discs, viewed from above, before the collision.



(a) Show that the speed of D immediately after the collision is 1.8 m s^{-1}, correct to 2 significant figures.

[4 marks]

(b) Find the speed of C immediately after the collision.

[2 marks]

(c) In fact the horizontal surface on which the discs are sliding is not smooth.

Explain how the introduction of friction will affect your answer to part **(b)**.

[2 marks]

Question 3

A disc, of mass 0.15 kg, slides across a smooth horizontal table and collides with a vertical wall which is perpendicular to the path of the disc.

The disc is in contact with the wall for 0.02 seconds and then rebounds.

A possible model for the force, F newtons, exerted on the disc by the wall, whilst in contact, is given by

$$F = kt^2(t-b)^2 \quad \text{for} \quad 0 \le t \le 0.020$$

where k and b are constants.

The force is initially zero and becomes zero again as the disc loses contact with the wall.

(a) State the value of b.

[1 mark]

(b) Find the magnitude of the impulse on the disc, giving your answer in terms of k.

[3 marks]

(c) The disc is travelling at 4 m s^{-1} when it hits the wall.

The disc rebounds with a speed of 2 m s^{-1}

Find k.

[3 marks]

Question 4

A lump of clay has mass 4 kg.

It is thrown so that it hits a vertical wall with a speed of 3 m s^{-1} and comes to rest, stuck to the wall.

It is assumed that the lump of clay takes 0.5 seconds to come to rest.

A possible model for the magnitude of the force, F newtons, exerted by the wall on the lump of clay is $F = at(2t-1)^2$ for $0 \le t \le 0.5$

(a) Find the value of a.

[5 marks]

(b) Find the kinetic energy lost as the lump of clay is brought to rest.

[2 marks]

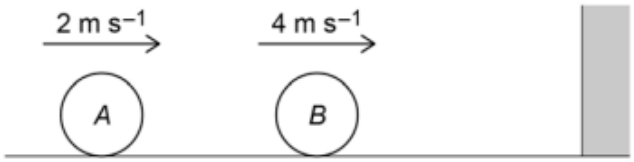
Question 5

Two spheres, *A* and *B*, of equal radii, move along a straight line on a smooth horizontal surface, in the same direction.

A vertical wall is perpendicular to the path of the spheres.

A moves at 2 m s^{-1} and *B* moves at 4 m s^{-1}.

The mass of *A* is 3 kg and the mass of *B* is 1 kg.



(a) The coefficient of restitution between sphere *B* and the wall is 0.75.

Find the speed of *B* after it collides with the wall.

[1 mark]

(b) The two spheres then collide.

The coefficient of restitution between the spheres is e.

(b) (i) Show that the velocity of A after the collision is $\frac{1}{4}(3-5e)$.

[4 marks]

(b) (ii) Find, in terms of e, the velocity of *B* after the collision.

[2 marks]

(b) (iii) If the direction of *A* is changed during the collision, what can be deduced about the value of e?

[2 marks]

Question 6

A ball of mass 0.3 kg is moving in a straight line with speed 4 m s^{-1}.

The ball collides with a wall perpendicular to its direction of motion and rebounds with speed 1 m s^{-1}.

Find the magnitude of the impulse, in Ns, exerted by the wall on the ball.

Circle your answer

[1 mark]

0.9 1.5 2.25 2.55

Question 7

Two smooth discs *A* and *B* have equal radii and are free to move on a smooth horizontal surface.

A and *B* have masses 4 kg and 2 kg respectively.

A moves with speed 3 m s^{-1} in a straight line and *B* moves in the same direction with speed 2 ms^{-1}.

A and *B* collide; the coefficient of restitution between *A* and *B* is e

(a) (i) Show that the speed of *A* immediately after the collision is

$$\frac{8-e}{3} \text{ m s}^{-1}$$

[4 marks]

(a) (ii) Find the speed of *B* immediately after the collision, in terms of e

[2 marks]

(b) The speed of *B* after the collision is k times the speed of *A*.

Find the maximum value of k

Fully justify your answer.

[3 marks]

Preview

Watch the vidoes by clicking on the link below. Make flashcards to help with the new knowledge. This will help you get a better headstart for next year.

Finding Determinant of 3 x 3 Matrix
Finding Inverse of 3 x 3 Matrix
De Moivre's Theorem
Euler's Form for Complex Numbers
Complex Roots of Unity
Vector Product Part 1
Vector Product Part 2

Use the AQA Spec document on OneNote to do some reading ahead about which topics are coming up next year.

Dan will love it if you have a list of questions to ask him when you return in September ☺

Optional Extension

Have a look at the “Further Risps” using:

http://www.s253053503.websitehome.co.uk/further-risps/further-risps-20-4-20.pdf

Try to expand your mathematical knowledge by trying at least **5** of them.

Here are some recommended ones that you might find interesting:

Further Risp 1: The Twizzle

Further Risp 2: The Spiral-Line Area

Further Risp 4: Series Arithmagon

Further Risp 5: Inverse Trig Triangles

Further Risp 8: Induction Number Theory

Further Risp 9: The Unexpected Group

Further Risp 12: Sketching Rational Functions

Further Risp 14: Three Simultaneous Equations

Further Risp 20: Matrix Arithmagon

Further Risp 22: The Vector Product

Further Risp 25: Trace Arithmetic

Further Risp 34: The Series Result

Solutions

Confidence Intervals

Question 1

3 (a)	Finds the width of the interval	AO1.1b	B1	$0.5(37.874 - 33.926) = 1.974$
	Use of $z \times \frac{6}{5} =$ UCL $-$ 35.9 or 35.9 $-$ LCL	AO3.1a	M1	$z \times \frac{6}{5} = 1.974$
	Finds a z-value and finds a relevant probability from that z-value	AO1.1a	M1	$z = 1.645$ Probability between -1.645 and $1.645 = 0.900$
	States correct confidence level	AO1.1b	A1	$k = 90$
	Total		**4**	
3 (b)	Clearly indicate that the width of the interval will be reduced (using any appropriate term for 'reduced')	AO2.2	E1	The width of the confidence interval will be reduced/smaller
	Total		**1**	

Question 2

Q	Marking Instructions	AO	Marks	Typical Solution
5(a)	Finds the correct sample mean.	AO1.1b	B1	$\overline{x} = \frac{963.2}{16} = 60.2$
	States the correct z-value.	AO1.1b	B1	z- value = 1.96
	Uses the correct structure of limits. Uses their $\overline{x}$ and z but must divide by $\sqrt{16}$ PI	AO1.1a	M1	$60.2 \pm \frac{1.96 \times 12.6}{\sqrt{16}}$
	Obtains correct values for the upper and lower limits.	AO1.1b	A1	= (54.0 , 66.4)
5(b)	States that the confidence interval contains 58.4, and infers that the claim is correct/justified/reasonable. (OE)	AO2.2b	E1	The confidence interval contains 58.4 so the claim seems reasonable.
	Total		**5**	

Contingency Tables

Question 1

<table>
<tr><th>Q</th><th>Marking Instructions</th><th>AO</th><th>Marks</th><th>Typical Solution</th></tr>
<tr><td rowspan="2">7(a)</td><td>Obtains one missing expected value eg
Low and Breed A $= \frac{21\times 27}{105}$</td><td>AO1.1a</td><td>M1</td><td rowspan="2"><table><tr><th></th><th>Low</th><th>Med</th><th>High</th></tr><tr><td>A</td><td>5.4</td><td>9.6</td><td>6</td></tr><tr><td>B</td><td>5.14</td><td>9.14</td><td>5.71</td></tr><tr><td>C</td><td>8.23</td><td>14.63</td><td>9.14</td></tr><tr><td>D</td><td>8.23</td><td>14.63</td><td>9.14</td></tr></table></td></tr>
<tr><td>Obtains all expected values.
A0 if integers</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td rowspan="5">(b)(i)</td><td>States both hypotheses using correct language</td><td>AO2.5</td><td>B1</td><td rowspan="5">H_0 Milk yield is independent of breed
H_1 Milk yield is not independent of breed 1 tail 1%

ts $= \sum \frac{(O-E)^2}{E} = 19.4$

χ^2 cv for 6 df = 16.81
($p = 0.00275$)

19.4 > 16.81
(0.00275 < 0.01)

Reject H_0 and conclude that there is evidence to suggest that milk yield is not independent of breed</td></tr>
<tr><td>States critical value (or p-value)</td><td>AO1.1b</td><td>B1</td></tr>
<tr><td>Evaluates the χ^2 test statistic by comparing the cv with the ts (or p-value with 0.01)</td><td>AO3.5a</td><td>R1</td></tr>
<tr><td>Infers H_0 rejected</td><td>AO2.2b</td><td>E1</td></tr>
<tr><td>Concludes correctly in context (conclusion should not be definite)</td><td>AO3.2a</td><td>E1</td></tr>
<tr><td rowspan="2">(b)(ii)</td><td>Considers $\frac{(O-E)^2}{E}$ or $(O-E)$ to identify largest sources of association
[Do not allow mark if no reference to why source selected]</td><td>AO2.4</td><td>E1</td><td rowspan="2">Largest sources of association
Breed A/High and Breed B/ Low.

Far more than expected Breed A cows observed to have high milk yield
or
Far more than expected Breed B cows observed to have low milk yield</td></tr>
<tr><td>Interprets main source(s) of association in context</td><td>AO3.2a</td><td>E1dep</td></tr>
<tr><td></td><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

Question 2

<table>
<tr><td>6 (a)</td><td>Clear indication that the data is from a random sample CAO</td><td>AO3.5b</td><td>B1</td><td>Must be data from a random sample</td></tr>
<tr><td></td><td>Total</td><td></td><td>1</td><td></td></tr>
<tr><td rowspan="2">6 (b)(i)</td><td>Obtains one missing expected value
eg for Fail/0–9
$\frac{59\times123}{200}=36.3$</td><td>AO1.1a</td><td>M1</td><td rowspan="2"><table><tr><td></td><td>0-9</td><td>10-19</td><td>20+</td></tr><tr><td>P</td><td>36.3</td><td>45.5</td><td>41.2</td></tr><tr><td>F</td><td>22.7</td><td>28.5</td><td>25.8</td></tr></table></td></tr>
<tr><td>Obtains all four values, must all be stated to nearest 0.1 for the A1 mark</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td></td><td>Total</td><td></td><td>2</td><td></td></tr>
</table>

6 (b)(ii)	States both hypotheses using correct terminology. Accept use of independence or association	AO2.5	B1	H_0: No association between test result and number of lessons taken H_1: There is an association between test result and number of lessons taken
	States the correct critical value (or p-value)	AO1.1b	B1	No of df $= (3-1)(2-1) = 2$ χ^2 value for 2 df at 1% level $= 9.21$
	Evaluates the χ^2 test statistic by comparing the critical value with the given value of the test statistic (or p-value with 0.01) Comparison needs to be seen	AO3.5a	R1	As $10.0 > 9.21$
	States inference that H_0 is rejected	AO2.2b	E1	Reject H_0
	Concludes in context (not too definitive eg just stating 'there is an association…')	AO3.2a	E1	The evidence suggests that there is an association between the test result and the number of lessons taken
	Total		**5**	
6 (b)(iii)	Any clear explanation that fully justifies that Wendy's claim is incorrect, using algebra or a numerical example from a cell as comparison	AO2.3	E1	$\frac{(42-36.3)^2}{36.3} = 0.895$ When doubled $\frac{(84-72.6)^2}{72.6} = 1.790$ So Wendy's claim is incorrect
	Total		**1**	

Question 3

<table>
<tr><td rowspan="9">3(a)</td><td>States correct null and alternative hypotheses.</td><td>AO2.5</td><td>B1</td><td rowspan="9">H_0: There is no association between age and drink taken with breakfast

H_1: There is an association between age and drink taken with breakfast

Test at the 10% significance level
Degrees of freedom = 2 x 2 =4
χ^2 critical value = 7.78

Expected frequencies:
<table><tr><td></td><td>Tea</td><td>Coffee</td><td>FJ/W</td><td></td></tr><tr><td><25</td><td>45</td><td>63</td><td>42</td><td>150</td></tr><tr><td>25-50</td><td>60</td><td>84</td><td>56</td><td>200</td></tr><tr><td>>50</td><td>45</td><td>63</td><td>42</td><td>150</td></tr><tr><td></td><td>150</td><td>210</td><td>140</td><td>500</td></tr></table>
$\Sigma\frac{(O_i-E_i)^2}{E_i}=\frac{(45-16)^2}{45}+\ldots$
$=18.8+\ldots$

18.8+… > 7.78

Hence reject H_0

There is significant evidence of an association between age and breakfast drink.</td></tr>
<tr><td>Clearly states significance level before any calculations are presented.</td><td>AO2.1</td><td>B1</td></tr>
<tr><td>Calculates expected frequencies.</td><td>AO1.1a</td><td>M1</td></tr>
<tr><td>Calculates at least six expected frequencies correctly.</td><td>AO1.1b</td><td>A1</td></tr>
<tr><td>Calculates χ^2 statistic.

Note that typically only one or two values are required to make the appropriate comparison with a critical value.</td><td>AO1.1a</td><td>M1</td></tr>
<tr><td>Finds correct critical value for their significance level.</td><td>AO1.1b</td><td>B1</td></tr>
<tr><td>Compares their correct χ^2 statistic to their critical value and infers that H_0 is rejected.</td><td>AO2.2b</td><td>B1</td></tr>
<tr><td>Completes test and concludes correctly in context.</td><td>AO3.2a</td><td>E1</td></tr>
<tr><td>3(b)</td><td>Explains that the expected frequency for e.g. tea and 16-25 would be less than 5, which is not valid for a χ^2 test. (so Celia's idea is not good.)</td><td>AO3.5b</td><td>E1</td><td>The χ^2 test will not be valid if any expected frequency is less than 5. For tea and 16-25, expected frequency is 4.5, which is less than 5, so Celia's suggestion is not a good idea</td></tr>
<tr><td></td><td>Total</td><td></td><td>9</td><td></td></tr>
</table>

Collisions

Question 1

2	Circles correct answer.	AO1.1b	B1	$I = \frac{1}{2} \times 0.8 \times 4 = 1.6\ \text{Ns}$

Question 2

5(a)	Forms an equation using conservation of momentum.	AO1.1a	M1	CoM $2 \times 4 - 2 \times 3 = 2v_C + 3v_D$
	Forms an equation using coefficient of restitution.	AO1.1a	M1	$2v_C + 3v_D = 2$ Newton's law of restitution
	Obtains two correct equations.	AO1.1b	A1	$v_C - v_D = -0.6(-2-4)$
	Completes a rigorous argument using both conservation of energy and the coefficient of restitution to find speed of D to the specified accuracy. Only award if they have a completely correct solution, which is clear, easy to follow and contains no slips.	AO2.1	R1	$v_C - v_D = -3.6$ $5v_D = 9.2$ $v_D = 1.84$ $= 1.8\ \text{m s}^{-1}$ to 2 sf
5(b)	Forms equation to find velocity of C	AO1.1a	M1	$1.84 - v_C = 3.6$ $v_C = -1.76$ Speed of $C = 1.8\ \text{m s}^{-1}$ to 2 sf
	Obtains correct speed for C.	AO1.1b	A1	
5(c)	Gives a valid explanation (eg collision is instantaneous, no distance travelled, no work done, no energy lost to friction during collision, etc)	AO2.4	E1	The introduction of friction will not affect my answer to **(b)** because the collision is instantaneous.
	Therefore answer to part **(b)** is not affected by the introduction of friction. (depends on E1 above)	AO2.2a	R1	
	Total		**8**	

Question 3

7(a)	Deduces correct value for b.	AO2.2a	B1	$b = 0.02$
7(b)	Forms an integral to find the impulse.	AO3.4	M1	$I = \int_0^{0.02} kt^2(t-0.02)^2\,\mathrm{d}t$
	Integrates terms and uses limits or uses a calculator for definite integral (PI)	AO1.1a	M1	$= k\int_0^{0.02} (t^4 - 0.04t^3 + 0.0004t^2)\,\mathrm{d}t$
	Obtains correct value for impulse. (AWRT 1.1×10^{-10})	AO1.1b	A1	$= k\left[\frac{t^5}{5} - \frac{t^4}{100} + \frac{t^3}{7500}\right]_0^{0.02}$ $= k \times 1.07 \times 10^{-10}$ Ns
7(c)	Uses 'impulse equals change in momentum' to form an equation, with 'their' impulse from **(a)**.	AO3.4	M1	$k \times 1.07 \times 10^{-10} = \lvert 0.15 \times 4 - 0.15 \times (-2) \rvert$
	Obtains a correct equation for 'their' impulse.	AO1.1b	A1F	$k = \frac{0.9}{1.07 \times 10^{-10}}$
	Obtains the correct value for k. CAO	AO1.1b	A1	$= 8.4 \times 10^9$
	Total		**7**	

Question 4

6 (a)	Forms an integral to find the impulse	AO3.4	M1	$\int_0^{0.5} at(2t-1)^2 \mathrm{d}t$ $= a\int_0^{0.5}\left(4t^3-4t^2+t\right)\mathrm{d}t$ $= \frac{a}{48}$ $4\times 3 = \frac{a}{48}$ $a = 576$
	Correctly evaluates 'their' integral	AO1.1b	A1	
	Uses $I = mv - mu$	AO1.1a	M1	
	Uses impulse to form an equation to find a	AO3.4	M1	
	Obtains correct value for a	AO1.1b	A1	
	Total		**5**	
6 (b)	Uses KE formula	AO1.1a	M1	KE lost $= \frac{1}{2}\times 4\times 3^2$ $= 18\ \mathrm{J}$
	Obtains correct energy	AO1.1b	A1	
	Total		**2**	

Question 5

Q	Marking instructions	AO	Marks	Typical solution
8 (a)	Uses law of restitution	AO1.1b	B1	$v = 0.75 \times 4 = 3$
	Total		**1**	
8 (b)(i)	Applies principle of conservation of momentum	AO1.1a	M1	$3 \times 2 + 1 \times (-3) = 3v_A + v_B$ $3 = 3v_A + v_B$ $v_A - v_B = -e(2-(-3))$ $v_A - v_B = -5e$ $v_B = v_A + 5e$ $3 = 3v_A + v_A + 5e$ $v_A = \frac{1}{4}(3-5e)$
	Applies Newton's law of restitution	AO1.1a	M1	
	Solves 'their' equations to find the velocity for *A*	AO1.1a	M1	
	Obtains correct velocity for *B*	AO2.1	A1	
	Total		**4**	
8 (b)(ii)	Substitutes velocity of *A* to find velocity *B*	AO1.1a	M1	$v_B = \frac{1}{4}(3-5e)+5e$ $v_B = \frac{3}{4}(1+5e)$
	Obtains correct velocity for *A*	AO1.1b	A1	
	Total		**2**	
8 (b)(iii)	Forms inequality based on velocity of *A*	AO2.2a	M1	$3-5e<0$ $e > \frac{3}{5}$
	Obtains correct inequality for e	AO1.1b	A1	
	Total		**2**	

Question 6

1	Circles correct answer.	AO1.1b	B1	1.5

Question 7

8(a)(i)	Forms a conservation of momentum equation.	AO1.1a	M1	$16 = 4v_A + 2v_B$
	Obtains a correct equation.	AO1.1b	A1	$v_B - v_A = e$
	Uses coefficient of restitution to create a second correct equation.	AO1.1b	B1	$16 = 4v_A + 2(v_A + e)$
	Completes a rigorous argument to obtain the correct expression for the speed of A.	AO2.1	R1	$16 = 6v_A + 2e$ $v_A = \frac{8-e}{3}$
8(a)(ii)	Substitutes the speed of A back into either of their equations, or eliminates v_A from their original equations.	AO1.1a	M1	$16 = 4(v_B - e) + 2v_B$ $16 = 6v_B - 4e$ $v_B = \frac{8+2e}{3}$
	Obtains correct speed for B.	AO1.1b	A1	
8(b)	Explains that k is a maximum when e is a maximum.	AO2.4	E1	$k = \frac{8+2e}{8-e}$ k is maximum when $e = 1$
	Forms an expression for k	AO1.1a	M1	
	Calculates correct value of k	AO1.1b	A1	$k = \frac{10}{7}$
	Total		**9**	